Driven Disordered Systems

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ICTP - SAIFR, São Paulo, 09/05/2025

Driven Disordered Systems

Agenda

L1. The **depinning** transition of elastic interfaces driven in disordered media and the **creep** motion at low driving

L2. The **yielding** transition of amorphous solids under deformation and associated **criticality** and **avalanche** statistics

L3. **Common framework** for depinning and yielding, analogies and **recent endeavors** in the field

L3. Common frameworks for yielding and depinning and recent endeavours

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L3. Common frameworks for yielding and depinning and recent endeavours

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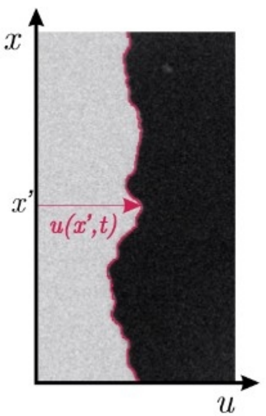
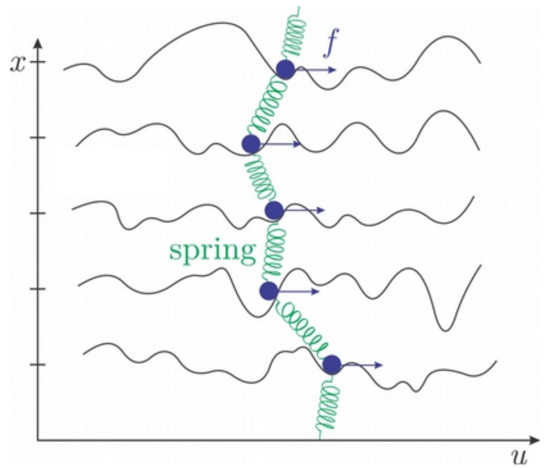
Unifying frameworks of depinning and yielding: models, flowcurves and effective noise

Unified description: equivalent description of elastoplastic models and elastic manifolds on disordered media.

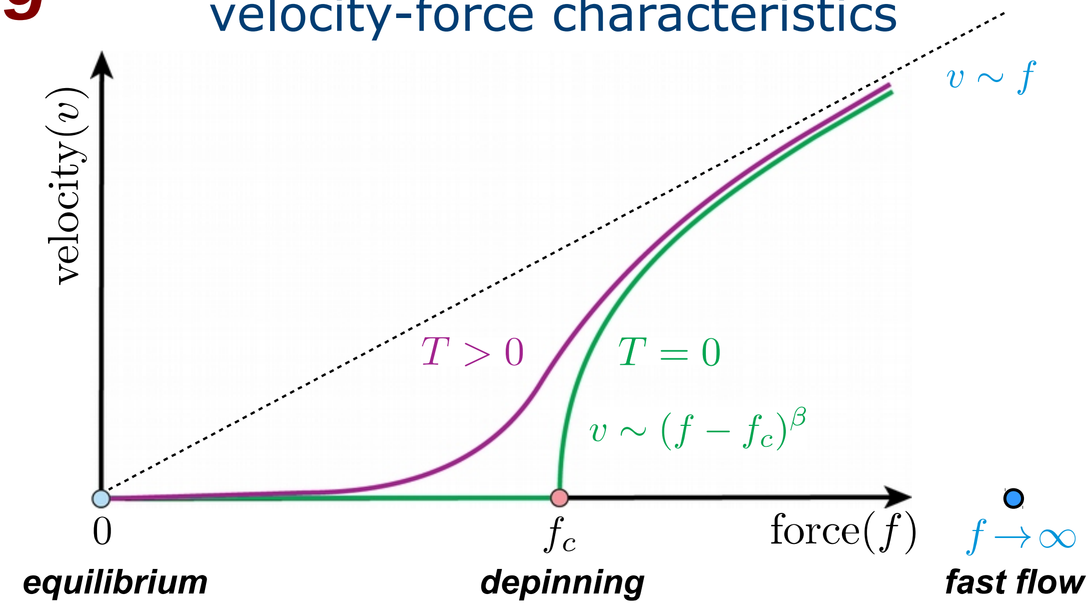
Unified problems: interpretation of yielding as an effective "one-particle" problem and its shared features with fully-connected mean-field depinning.

Depinning





velocity-force characteristics



EEF et al. C. R. Physique **14**, 641 (2013)

e.g., the Quenched-Edwards-Wilkinson elastic line:

$$\gamma \partial_t u(x,t) = \underbrace{c\partial_x^2 u(x,t)}_{\text{elastic interaction}} + \underbrace{f_{\mathrm{p}}(u,x)}_{\text{pinning force}} + \underbrace{f}_{\text{external force}} + \underbrace{\eta_T(x,t)}_{\text{thermal noise}}$$

$$f_p = -\frac{d}{du} V_p(u,x) \longleftarrow \text{disorder potential}$$

When long-range elasticity:

$$c\partial_x^2 u(x,t) \to c \int \frac{[u(x,t) - u(x',t)]}{|x - x'|^{d+\alpha}} d^d x'$$

- **Critical exponents** depend on *α* and *d*
- For fully-coupled (*α≤0*), ***β*** also depends on the **type of disorder** (if it's "continuously differentiable" or not).

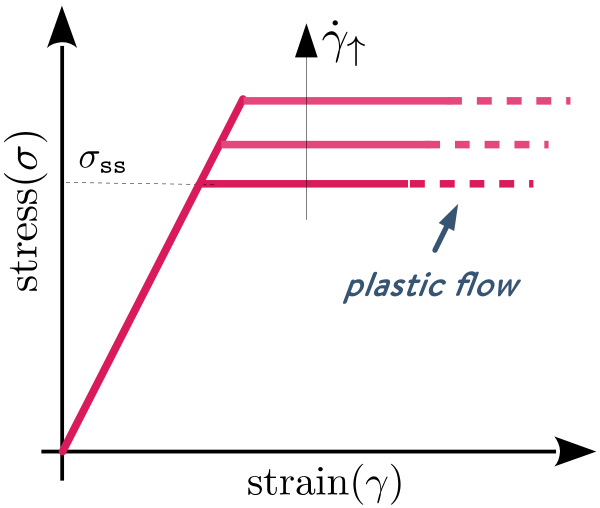
M. Kardar Phys. Reports (1998)

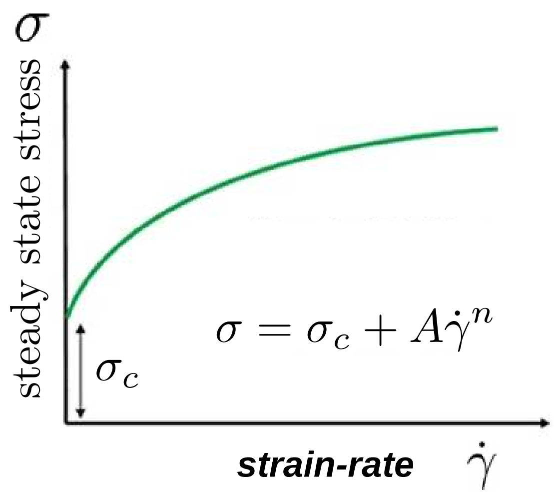
D. Fisher Phys. Reports (1998)

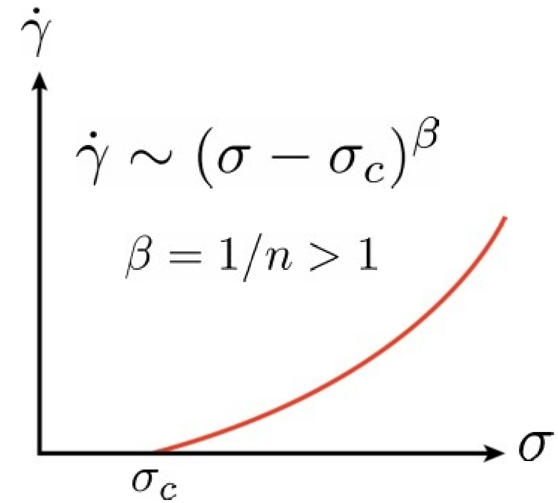
Yielding

Amorphous materials

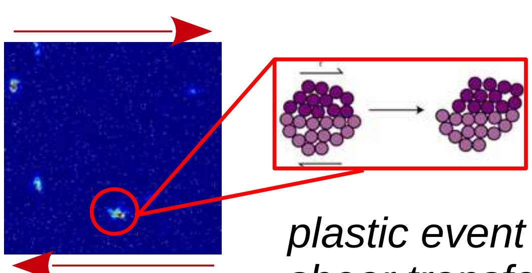
A. Nicolas et al. Rev. Mod. Phys. (2018)





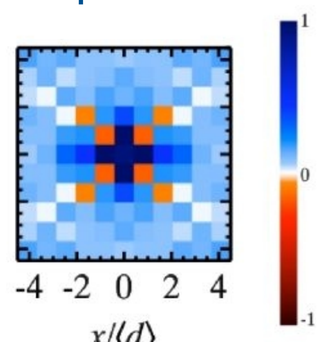


Local rearrangements + Medium elastic response



plastic event or *shear transformation*

Nicolas et. al EPJE (2014)



stress change (2D emulsion)
Desmond&Weeks PRL 2015

$$G(r,\theta) \sim \frac{\cos(4\theta)}{r^d}$$

Eshelby's propagator **for the stress redistribution**

J.D. Eshelby Proc.Roy.Soc. A (1957)

Coarse-grained Elasto-Plastic Models (EPM)

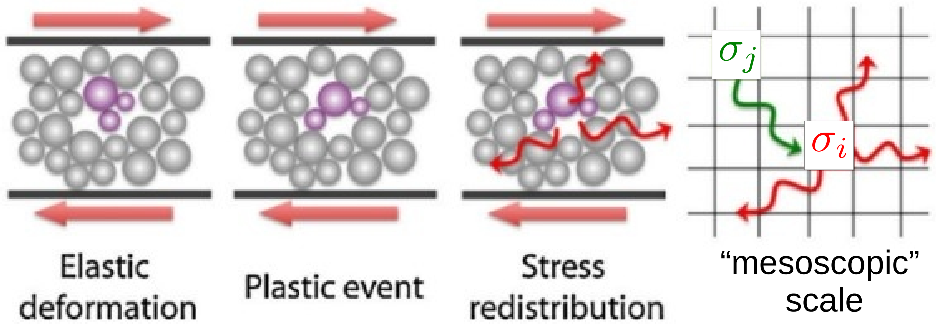


Fig. credit: Bocquet *et al*. PRL **103**, 036001 (2009)

$$\partial_t \sigma_i(t) = \underbrace{\mu\dot{\gamma}^{\mathtt{ext}}}_{\text{global loading imposed strain rate}} - \underbrace{g_0 n_i(t)\frac{\sigma_i(t)}{\tau}}_{\text{exponential stress decay when fludized}} + \underbrace{\sum_{j\neq i} G(i,j) n_j(t)\frac{\sigma_j(t)}{\tau}}_{\text{"mechanical noise" due to plastic activity elsewhere}}$$

global loading imposed strain rate

exponential stress decay when fludized

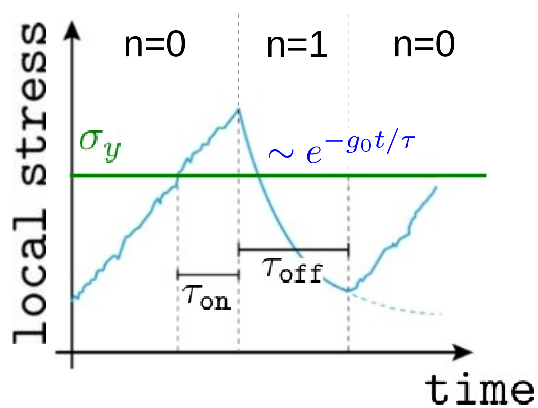
"**mechanical noise**" due to plastic activity elsewhere

+ Dynamical **rules** for the local "state" n_i

$n_i(t) = 0$ locally elastic

$n_i(t) = 1$ locally plastic

$$\boxed{n_i : \begin{cases} 0 \to 1 \text{ typically when } \sigma_i > \sigma_{y_i} \\ 1 \to 0 \text{ e.g., after a time } \tau_{\mathtt{off}} \end{cases}}$$



$$g_0 \equiv -G_{ii} > 0$$

$$\boxed{G_{ij}^{2D} \propto \frac{\cos(4\theta_{ij})}{r_{ij}^2}} \quad (i \neq j)$$

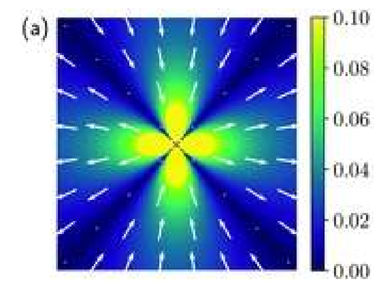
Eshelby **propagator** $G \sim \dfrac{1}{r^d}$

Yielding model++: manifold on disordered potential

Starting w/ a tensorial description $E_{\rm elast.} = \int d^2\mathbf{r}\ \left(Be_1^2 + \mu e_2^2 + \mu e_3^2\right)$

Allowing for plasticity: $E_{plast.} = \int d^2\mathbf{r}\ \left(Be_1^2 + \mu e_2^2 + V_{\underline{x}}[\mathbf{r}, e_3]\right)$

+ Saint-Venant constraints



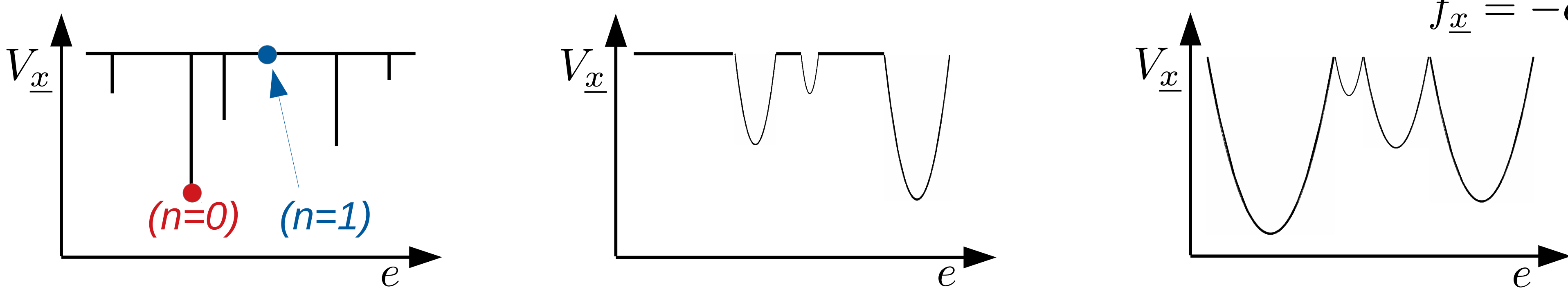
X. Cao et al, Soft Matt.(2018)

Assuming (overdamped dynamics), that e_1 and e_2 relax much faster than $e \equiv e_3$

“Elastic manifold” e(**r**) (with long-range interactions)

$$\partial_t e(\mathbf{r}, t) = \int \underbrace{d^2\mathbf{r}' G(\mathbf{r} - \mathbf{r}') e(\mathbf{r}')}_{\text{“Eshelby” interaction}} + \underbrace{f_{\underline{x}}(\mathbf{r}, e)}_{\text{pinning force}} + \underbrace{\sigma_{\rm ext}}_{\text{applied stress}}$$

$f_{\underline{x}} = -\partial_\varepsilon V_{\underline{x}}$



more “EP-like” → more typical

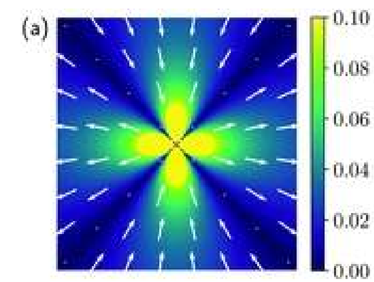
E.A. Jagla, PRE (2007)&(2020), C. Liu et al. JCP (2022)

Yielding model++: manifold on disordered potential

Starting w/ a tensorial description $E_{\text{elast.}} = \int d^2\mathbf{r}\ \left(Be_1^2 + \mu e_2^2 + \mu e_3^2\right)$

Allowing for plasticity: $E_{plast.} = \int d^2\mathbf{r}\ \left(Be_1^2 + \mu e_2^2 + V_{\underline{x}}[\mathbf{r}, e_3]\right)$

+ Saint-Venant constraints $\epsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$



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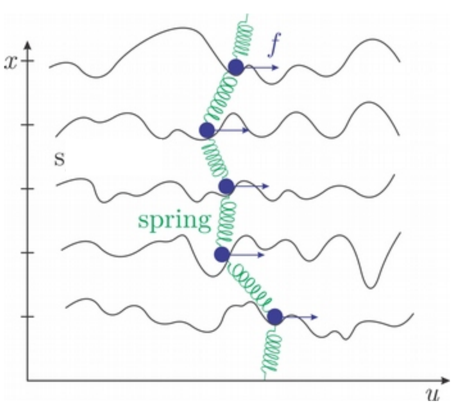
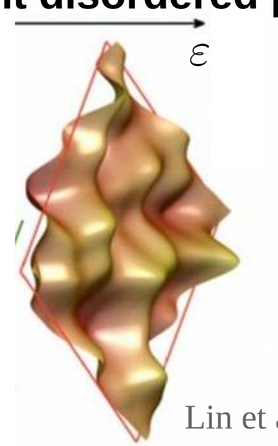
Each site w/ **independent disordered potential**





Lin et al. PNAS '14

By construction: no "states n_i", **no memory!**

Dynamics can be derived from a Hamiltonian and be a Markovian process

Disordered potential can be quenched or change dynamically after each basin jump

E.A. Jagla, PRE (2007)&(2020), C. Liu et al. JCP (2022)

“Hamiltonian” model for yielding

Evolution of the local strain *e(**r**,t)* on a mesh

$$\frac{\partial e_i}{\partial t} = \underbrace{-\frac{dV_i}{de_i}}_{\text{Pinning force}} + \overbrace{\sum_j G_{ij} e_j}^{\text{“Eshelby” interaction}} + \underbrace{\sigma}_{\text{applied stress}} + \overbrace{\sqrt{T}\xi_i(t)}^{\text{Langevin noise}}$$

$$\langle \xi_i(t) \rangle = 0$$

$$\langle \xi_i(t) \xi_j(t') \rangle = 2\delta(t - t')\delta_{ij}.$$

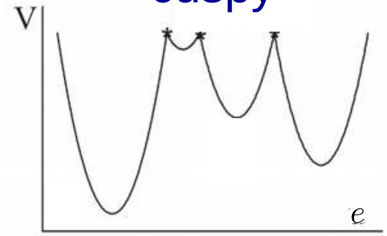
Eshelby **propagator**

$$\boxed{G_{ij}^{2D} \propto \frac{\cos(4\theta_{ij})}{r_{ij}^2}}$$

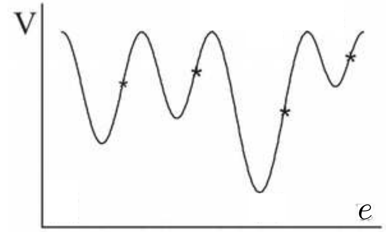
Measured strain-rate $\dot{\gamma} \equiv \overline{\frac{de_i}{dt}} = -\overline{\frac{dV_i}{de_i}} + \sigma$

V_i are disordered potentials, uncorrelated in space. We consider two different forms:

“cuspy”

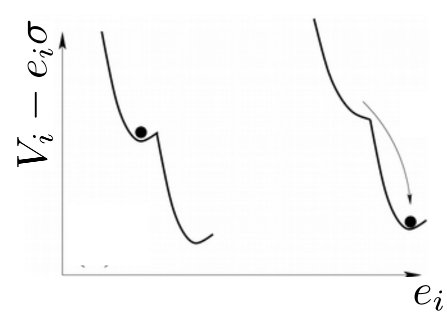


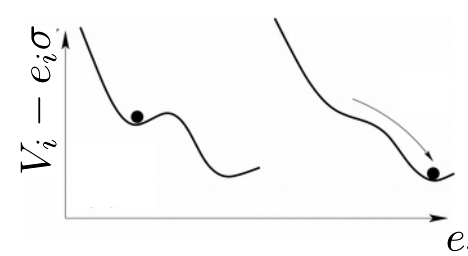
“smooth”



concatenation of parabolic or sinusoidal pieces

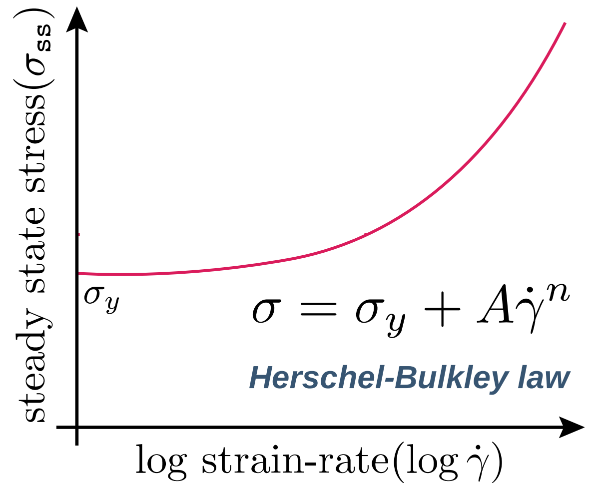
External driving σ tilts the potential, temperature helps to jump barriers

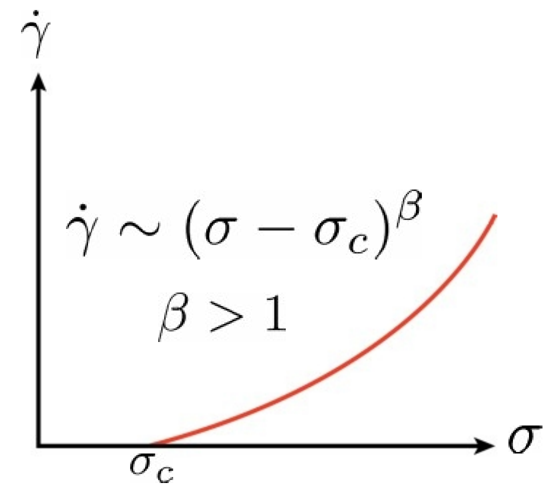




Each local jump to a new well is a plastic event

FLOWCURVES

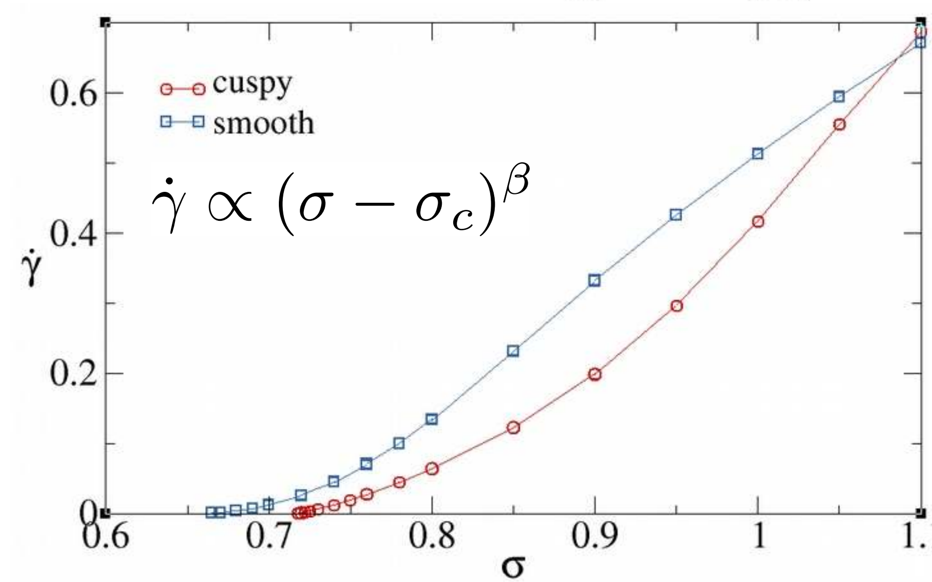

steady state stress(σ_{ss})
σ_y
$\sigma = \sigma_y + A\dot{\gamma}^n$
Herschel-Bulkley law
log strain-rate$(\log\dot{\gamma})$


$\dot{\gamma}$
$\dot{\gamma} \sim (\sigma - \sigma_c)^\beta$
$\beta > 1$
σ
σ_c

Flowcurves in the "Hamiltonian" model

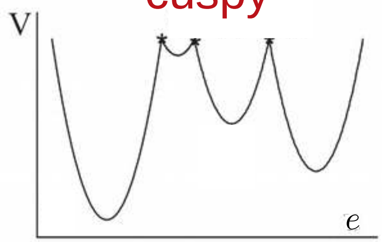
$$\frac{\partial e_i}{\partial t} = -\frac{dV_i}{de_i} + \sum_j G_{ij} e_j + \sigma$$

Measured strain-rate $\dot{\gamma} \equiv \frac{d\overline{e_i}}{dt} = -\overline{\frac{dV_i}{de_i}} + \sigma$

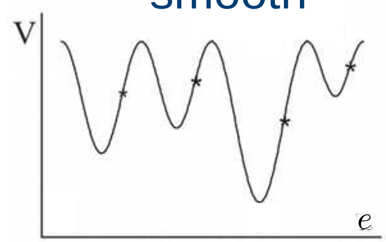


Consider two different disorder forms:

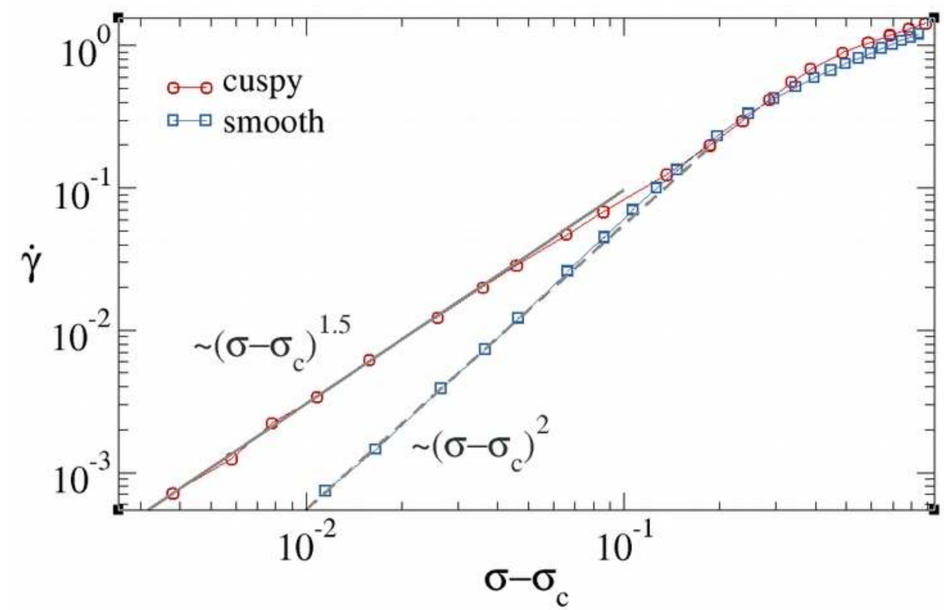
"cuspy"



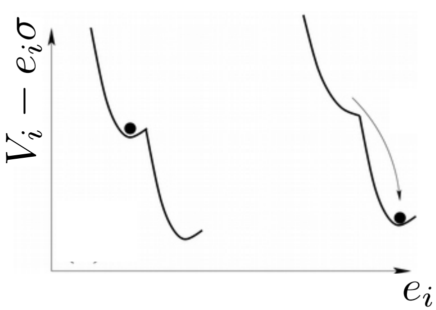
"smooth"

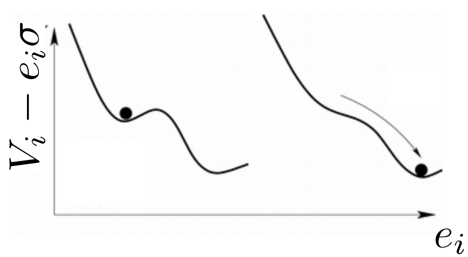


concatenation of parabolic or sinusoidal pieces



External driving σ tilts the potentials

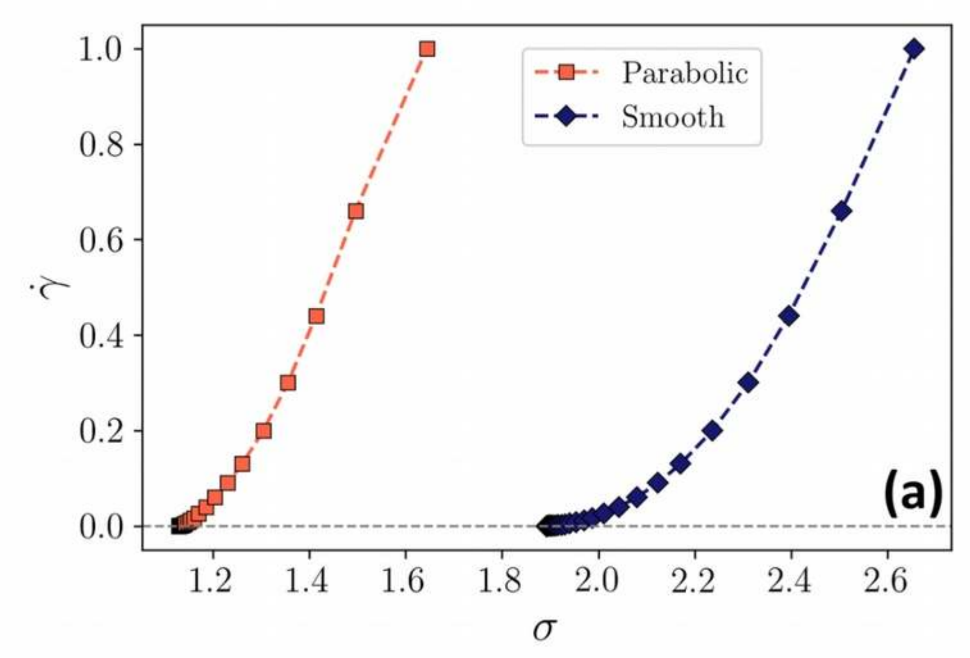


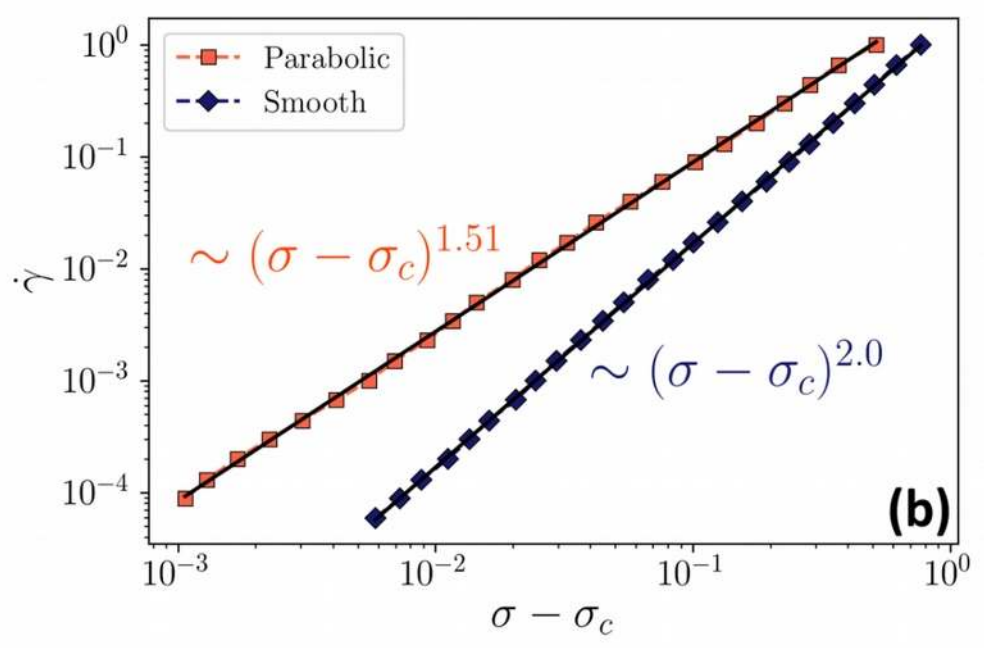


Each jump to a new well is a local "plastic event"

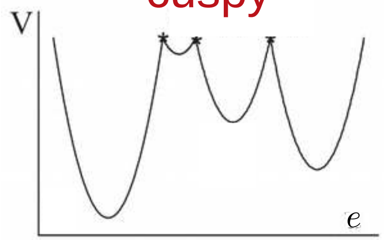
Flowcurves in the “Hamiltonian” model

Two relevant values for “beta”

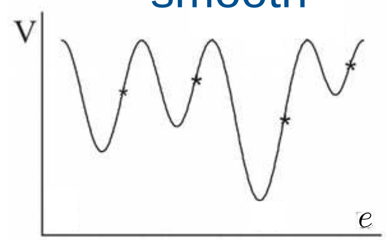




“cuspy”



“smooth”



Cuspy

$$\beta \simeq 1.5$$

Smooth

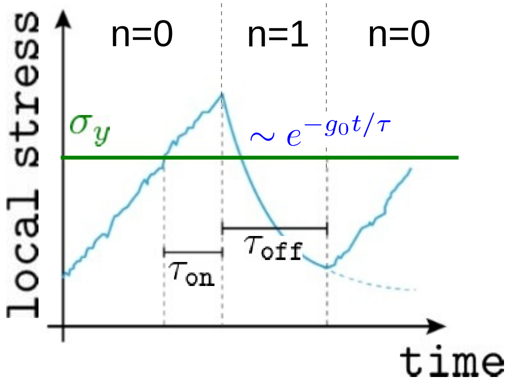
$$\beta \simeq 2$$

I Fernández Aguirre, EA Jagla PRE **98**, 013002 (2018)

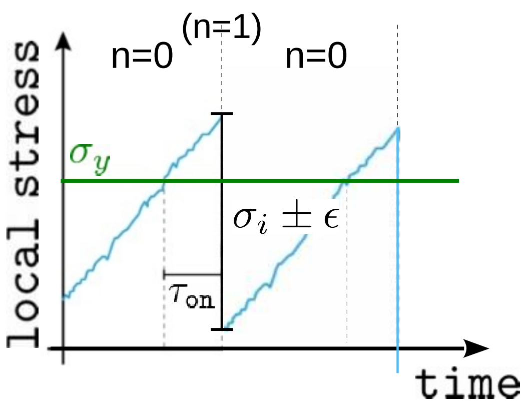
Elasto-Plastic Models (EPM) with stress-dependent rates

EEF & EA Jagla *Soft Matter* **15**, 9041 (2019)

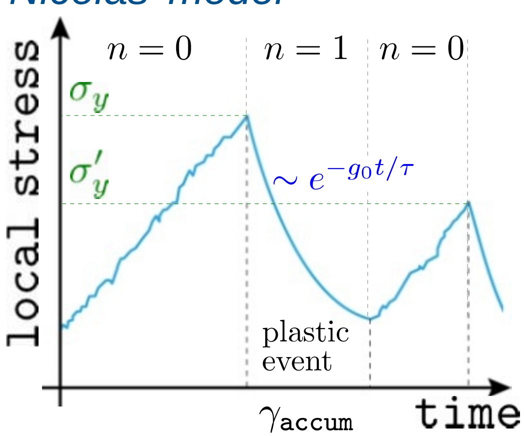
Picard's model



Lin's model



Nicolas' model



$$\partial_t \sigma_i(t) = \mu \dot{\gamma}^{\rm ext} - g_0 n_i(t) \frac{\sigma_i(t)}{\tau} + \sum_{j \neq i} G_{ij} n_j(t) \frac{\sigma_j(t)}{\tau}$$

$n_i :\ 0 \rightarrow 1$ when $\sigma_i > \sigma_i^y$ at a rate $\tau_{\rm on}^{-1}$

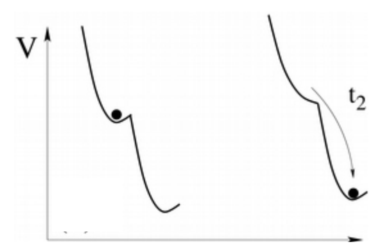
Stochastic rules for local yielding:

(all classic cases)	(our proposal)
Uniform rate	*Progressive rate*
$\tau_{\rm on}^{-1} = \lambda = \mathtt{cst.}$	$\tau_{\rm on}^{-1} = \lambda(\sigma) \propto \sqrt{\sigma - \sigma_y}$

site is more likely to yield as it is more overloaded

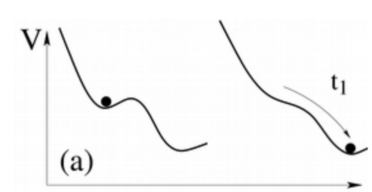
Rationale for the analogy with explicit disorder landscape

cuspy potential



$t_2 \sim \mathtt{cst.}$

smooth potential



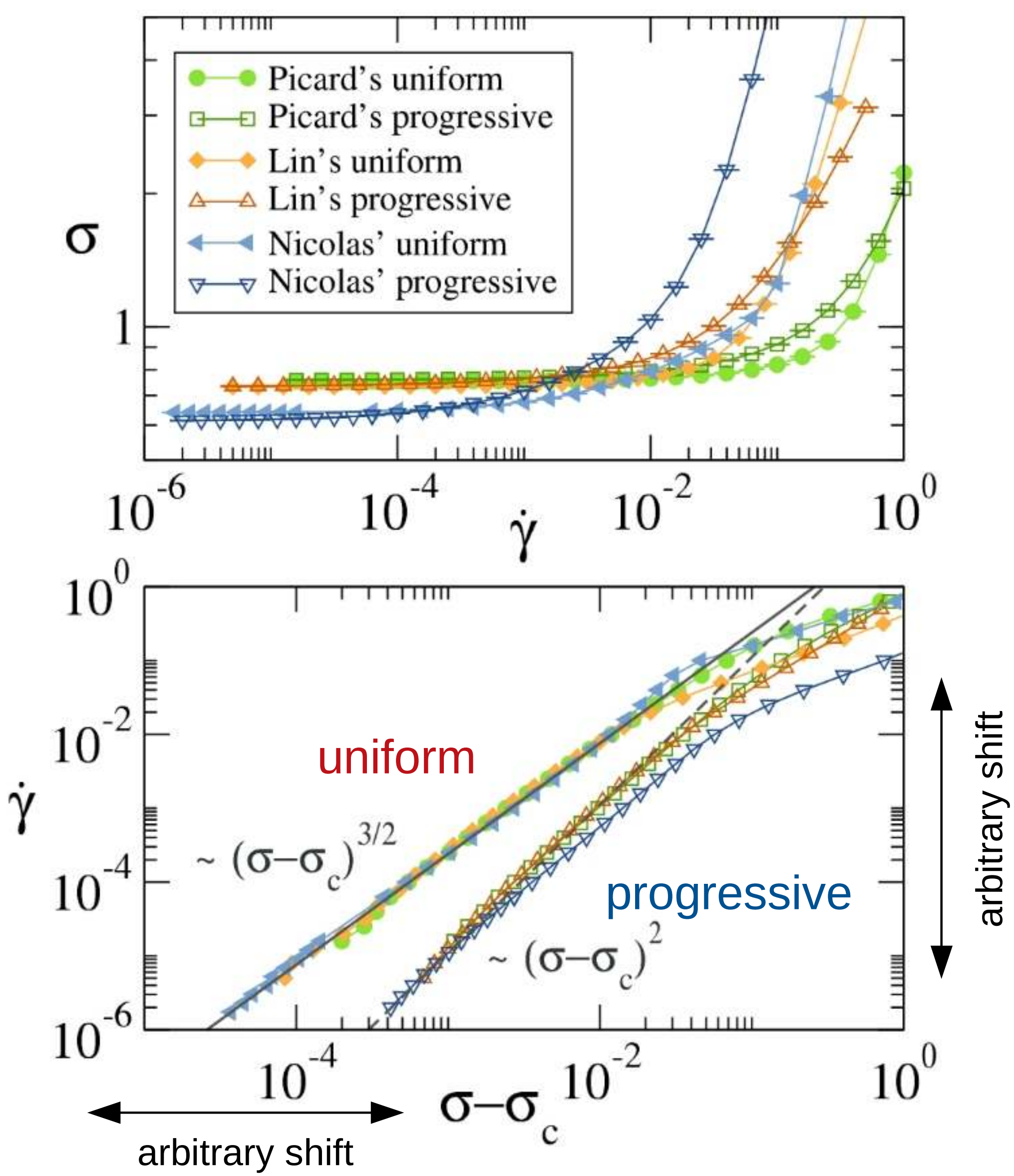
$t_1 \sim \delta\sigma^{-1/2}$

Flowcurves (β exponent)

d=2 T=0

We simulate (3x2=)**6 different** EPMs in *d=2*

EEF & EA Jagla *Soft Matter* **15**, 9041 (2019)



$$\dot\gamma \propto (\sigma - \sigma_c)^\beta$$

Uniform rate

$$\tau_{\texttt{on}}^{-1} = \lambda = \texttt{cst.}$$

$$\beta \simeq 1.5$$

Progressive rate

$$\tau_{\texttt{on}}^{-1} = \lambda(\sigma) \propto \sqrt{\sigma - \sigma_y}$$

$$\beta \simeq 2$$

β depends on the local yielding rule only!

(recall) In the Hamiltonian model:

Cuspy

$$\beta \simeq 1.5$$

Smooth

$$\beta \simeq 2$$

Flowcurve's β exponent and disorder type

Summary

***Yielding** in d=2*

$$\dot{\gamma} \propto (\sigma - \sigma_c)^\beta$$

Uniform rate or cuspy potential

$$\beta = 1.5$$

Progressive rate or smooth potential

$$\beta = 2$$

$$G \sim 1/r^{d+\alpha}$$

Notice that Eshelby is $\alpha = 0$

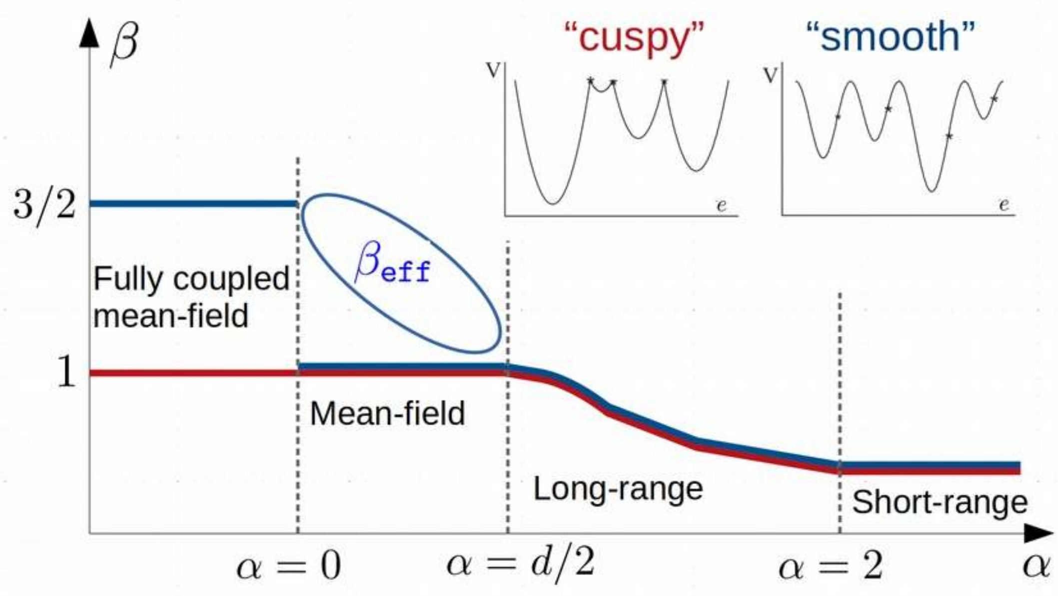
$$G(r,\theta) \sim \frac{cos(4\theta)}{r^d}$$

A difference of 0.5 in exponents between cuspy and smooth persists

***Depinning** of the elastic manifold*

When long-range: $F_{\mathtt{el}} = c \int \frac{[u(x,t) - u(x',t)]}{|x-x'|^{d+\alpha}} d^d x'$

$$G \sim 1/r^{d+\alpha}$$



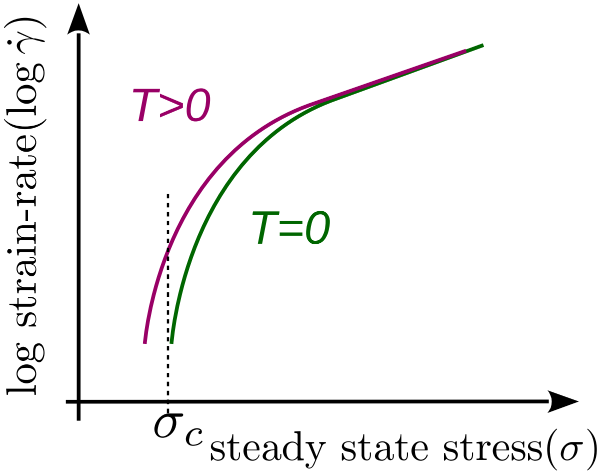
In fully-coupled depinning **two different β coexist**

A.B. Kolton and E.A. Jagla, PRE 98, 042111 (2018)
D.S. Fisher, Phys. Rev.p 301, 113 (1998)

1st observation:

The flowcurve exponent β of yielding in 2D depends on the type of disorder (or yielding rule), just as in fully-connected depinning

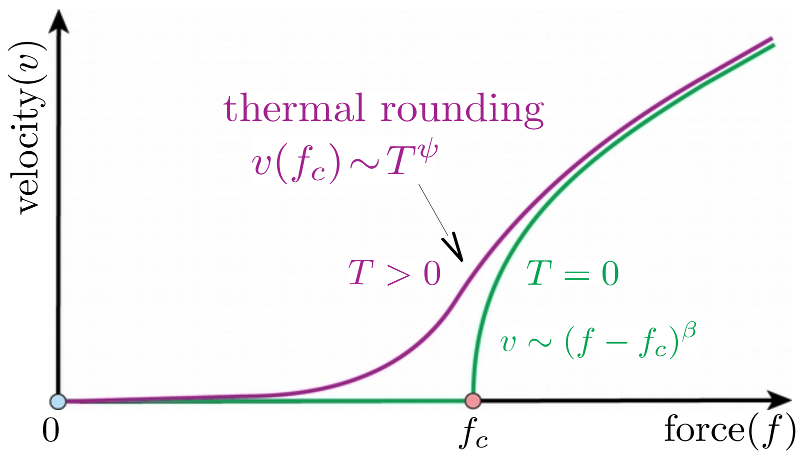
THERMAL ROUNDING



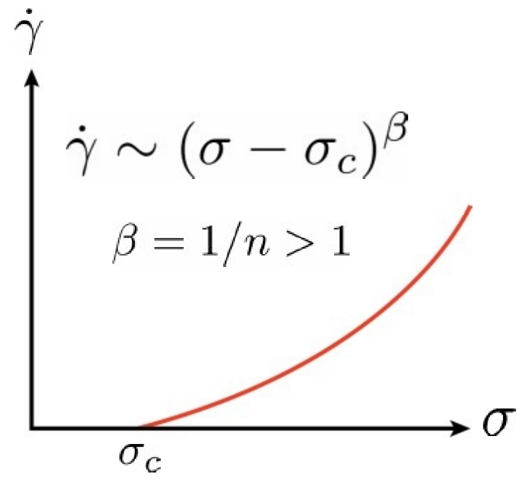
EEF, AB Kolton & EA Jagla, Phys. Rev. Materials **5**, 115602 (2021)

Thermal rounding scaling

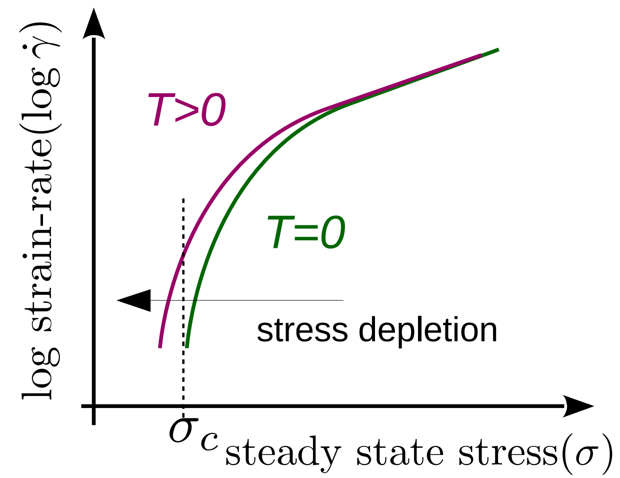
Depinning



Yielding



Better seen in log-lin...



In mean-field depinning*

$$\boxed{v = T^{\psi} \mathcal{F}((f - f_c)/T^{1/\alpha})}$$

$$\boxed{\dot{\gamma}(\sigma, T) = T^{\psi} \mathcal{G}((\sigma - \sigma_c)/T^{1/\alpha})}$$

following the analogy with a ferromagnetic transition

Right at f=f_c : $v(f_c, T) \sim T^{\psi}$

In the limit $T \rightarrow 0$ we expect $v \sim (f - f_c)^{\beta}$

so

$\mathcal{F}(x) \sim (x)^{\beta}$ and $\psi = \beta/\alpha$

$\psi = \beta/\alpha$ is the thermal rounding exp.

α describes how energy barriers vanish as we increase the stress

$$h_b \sim (\sigma_c - \sigma)^{\alpha}$$

$\alpha = 2$ for cuspy potentials

$\alpha = 3/2$ for smooth potentials

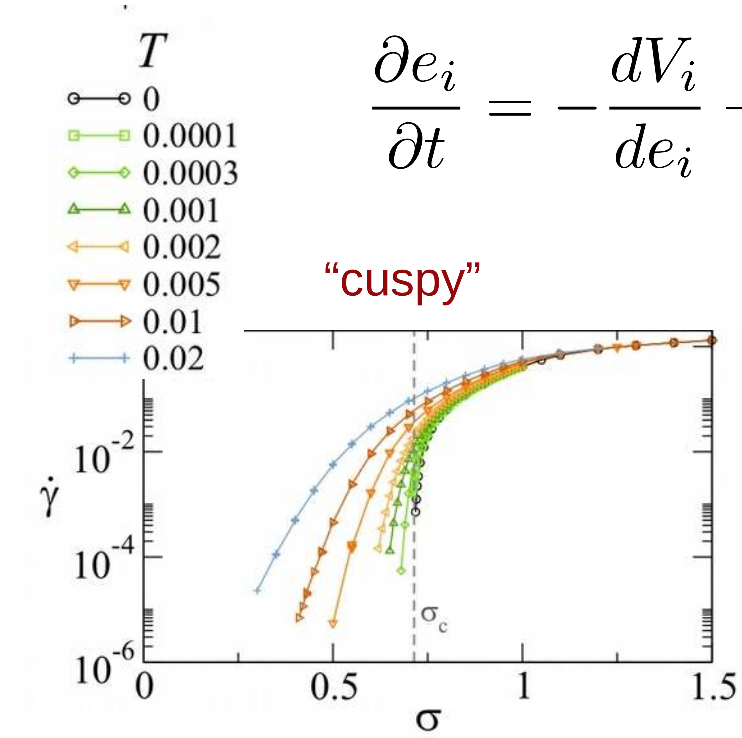
*[D.S. Fisher PRB 31, 1396 (1985)]

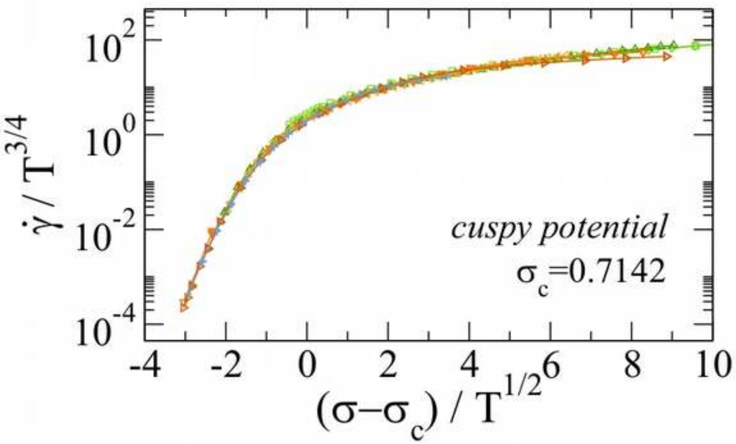
Thermal rounding: Hamiltonian model

EEF, AB Kolton & EA Jagla
Phys. Rev. Materials **5**, 115602 (2021)

$$\frac{\partial e_i}{\partial t} = -\frac{dV_i}{de_i} + \sum_j G_{ij} e_j + \sigma + \underbrace{\sqrt{T}\xi_i(t)}_{\text{Langevin noise}}$$

“cuspy”



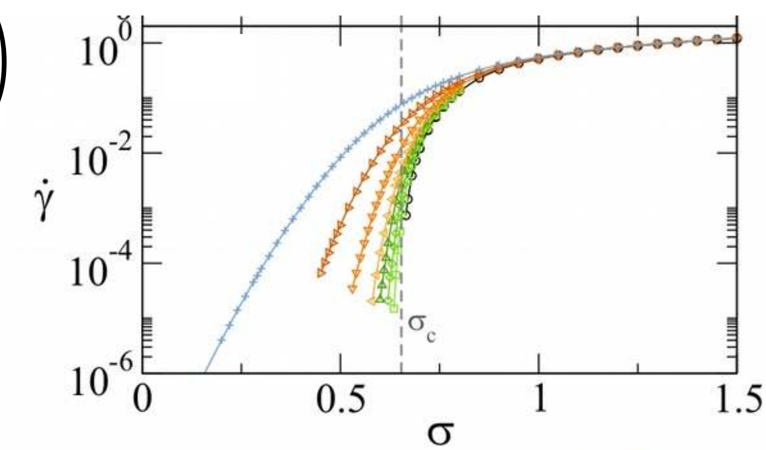


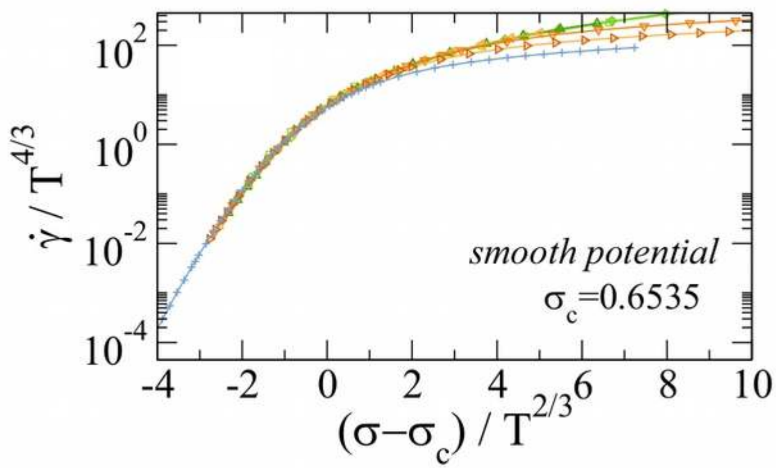
$$\frac{\dot\gamma(\sigma,T)}{T^{\psi}} = \mathcal{G}\left(\frac{\sigma-\sigma_c}{T^{1/\alpha}}\right)$$

The thermal rounding scaling works nicely around $\sigma=\sigma_c$ in both cases

(Temperatures to be compared with the reference energy ~1, typical height of local barriers)

“smooth”





$$\alpha = 2, \beta \simeq 3/2, \psi \simeq 3/4$$

$$\psi = \beta/\alpha$$

$$\alpha = 3/2, \beta \simeq 2, \psi \simeq 4/3$$

EP models with **thermal activation**

T>0

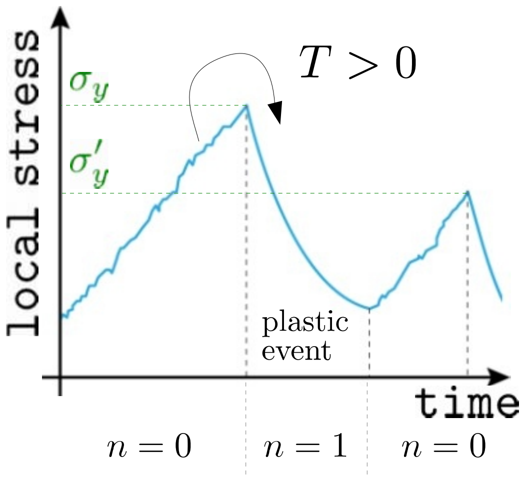
The e.o.m. remains identical

$$\partial_t \sigma_i(t) = \mu \dot{\gamma}^{\mathtt{ext}} - g_0 n_i(t) \frac{\sigma_i(t)}{\tau} + \sum_{j \neq i} G_{ij} n_j(t) \frac{\sigma_j(t)}{\tau}$$

We add the chance to locally fluidize by thermal activation while the stress is still **below** the threshold

Thermal model rules

$$n_i : \begin{cases} 0 \to 1 & \text{instantaneously} \quad \text{if} \quad \sigma_i \geq \sigma_{\mathrm{Y}} \\ 0 \to 1 & \text{with probability per unit time} \\ & \boxed{\exp(-(\sigma_{\mathrm{Y}i} - \sigma_i)^{\alpha}/T)} \quad \text{if} \quad \sigma_i < \sigma_{\mathrm{Y}} \\ 0 \leftarrow 1 & \text{at a rate } \tau_{\mathtt{off}}^{-1} \end{cases}$$



α plugged-in as a **new parameter** of the model

We restrict ourselves to the case of uniform rates $(\beta \simeq 3/2)$

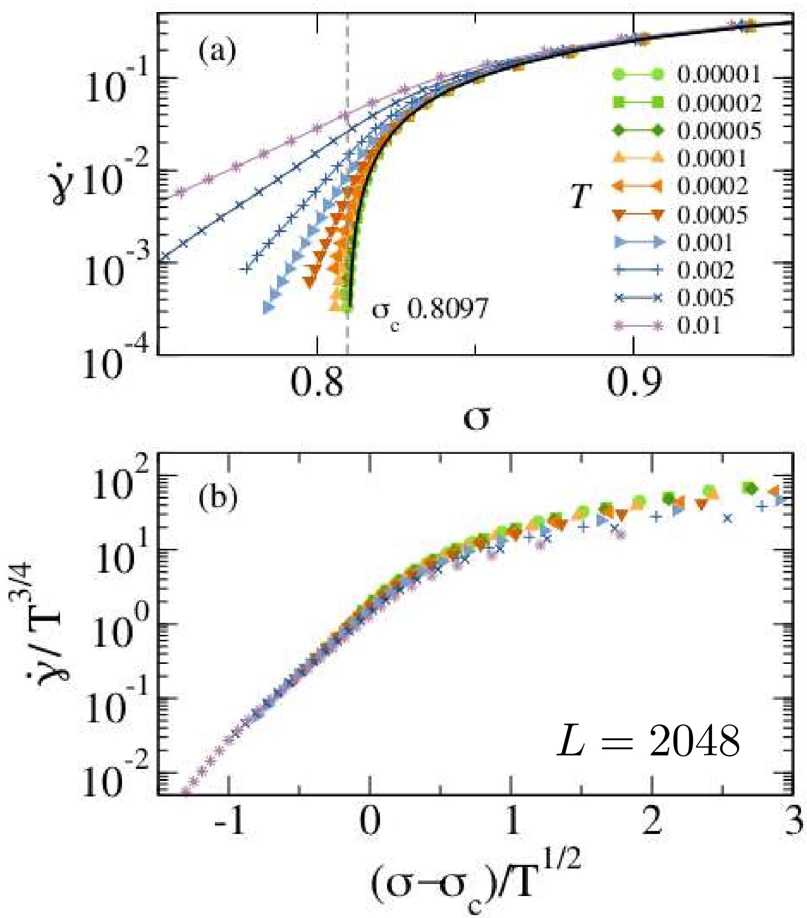
(progressive rate + activation is tricky)

Thermal rounding: EP models $\boxed{d=2}$ $\boxed{T>0}$

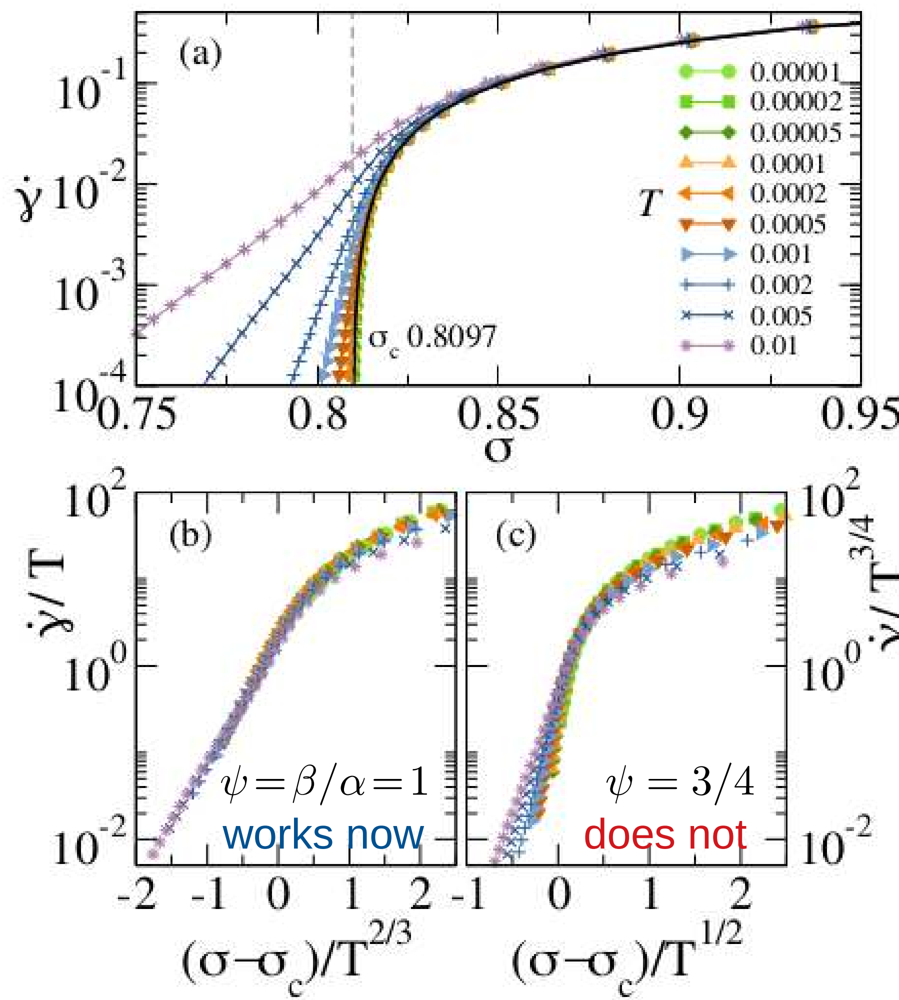
Uniform rates $\boxed{\beta \simeq 3/2}$

$$\frac{\dot{\gamma}(\sigma,T)}{T^{\psi}} = \mathcal{G}\left(\frac{\sigma-\sigma_c}{T^{1/\alpha}}\right)$$

$\alpha = 2$



$\alpha = 3/2$



$\alpha = 2, \psi = 3/4$ $\qquad \psi = \beta/\alpha$

Here α is a parameter, **decoupled** from β, but the scaling works well as soon as we use $\psi = \beta/\alpha$

EEF, AB Kolton & EA Jagla
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Thermal rounding scaling for the yielding transition

Summary

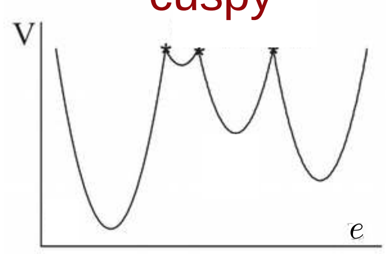
- The thermal rounding scaling **works with no corrections** for yielding in finite dimensions!

$$\frac{\dot{\gamma}(\sigma,T)}{T^{\psi}} = \mathcal{G}\left(\frac{\sigma-\sigma_c}{T^{1/\alpha}}\right)$$

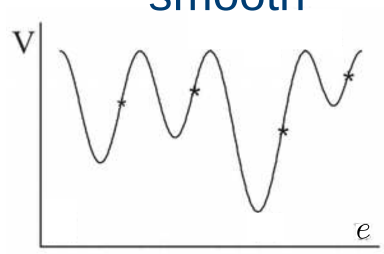
- **This is NOT the case* for usual depinning (short-range interactions)**
  - Numerical determination of ψ varies widely among different models
  - Universality questioned
  - Strong logarithmic corrections

Two relevant families of *linked* exponents

"cuspy"



"smooth"



	"cuspy"	"smooth"
	$\alpha = 2$	$\alpha = 3/2$
(d=2)	$\beta \simeq 3/2$	$\beta \simeq 2$
Then	$\psi \simeq 3/4$	$\psi \simeq 4/3$

*Several works:
Kolton&Jagla, PRE 102, 052120 (2020)
Bustingorry et al. EPL 81, 26005 (2007), Physica B: Condensed Matter 404, 444 (2009), PRE 85, 021144 (2012).

EEF, AB Kolton & EA Jagla
Phys. Rev. Materials **5**, 115602 (2021)

See also: M. Popović et al. PRE 104, 025010 (2021)

2^{nd} observation:

Standard “à la Fisher” thermal rounding scaling works just perfectly for yielding in 2D, just as in fully-connected depinning

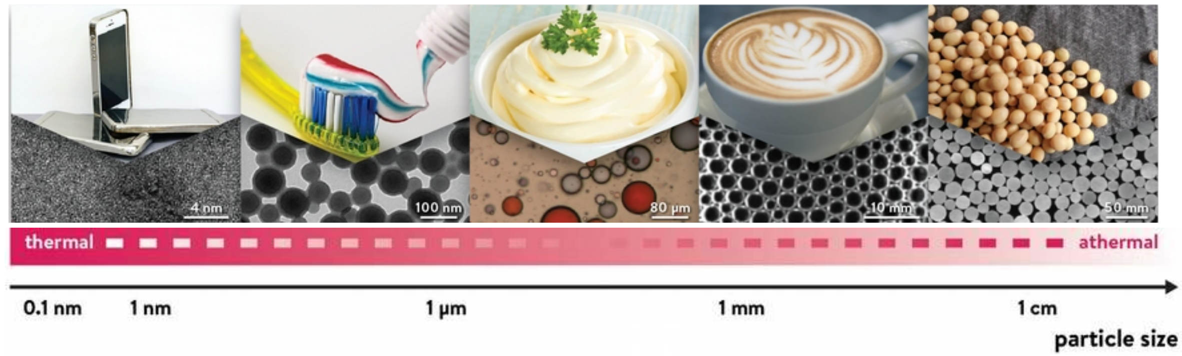
(

Yielding transition at finite temperatures

Is temperature relevant here?

In general not but…

Thermal effects (beyond intrinsic) could be relevant when elementary constituents<1μm



Rev. Mod. Phys. 90, 045006 (2018)

PRL **95,** 195501 (2005) PHYSICAL REVIEW LETTERS week ending 4 NOVEMBER 2005

A Universal Criterion for Plastic Yielding of Metallic Glasses with a $(T/T_g)^{2/3}$ Temperature Dependence

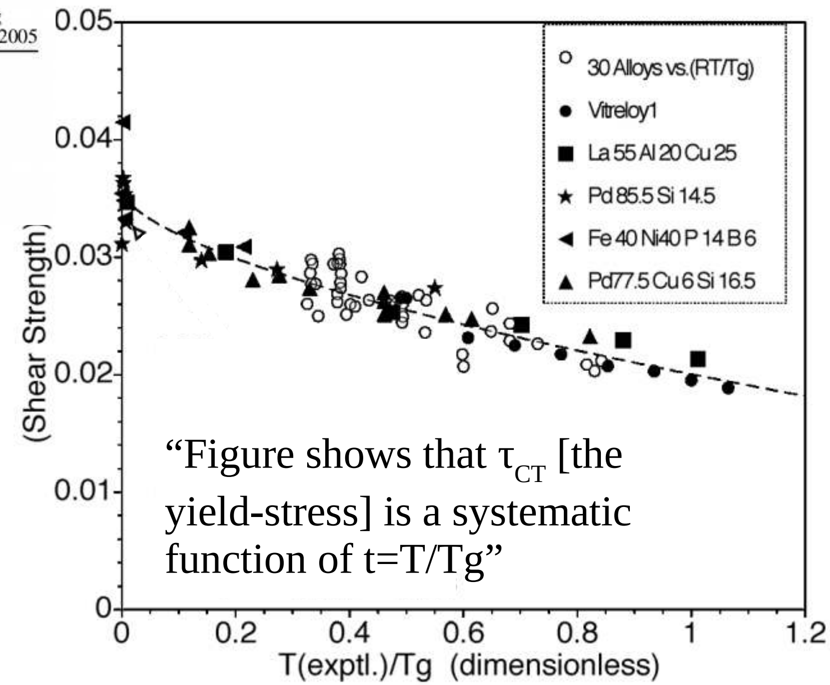
W. L. Johnson* and K. Samwer†

(~ 1k citations)

Empirically showed that the yield-strength [τ_{CT}] (stress at yielding) is a systematic function of T/Tg, **in more than 30 different metallic glasses**.

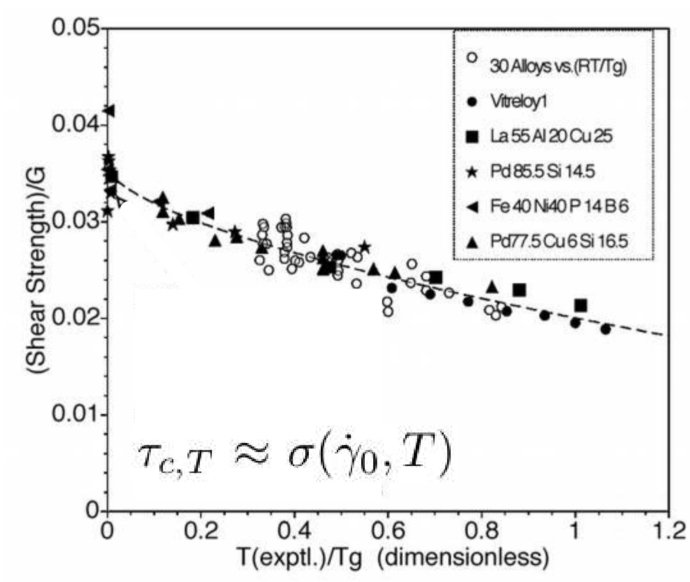
Strong universality!

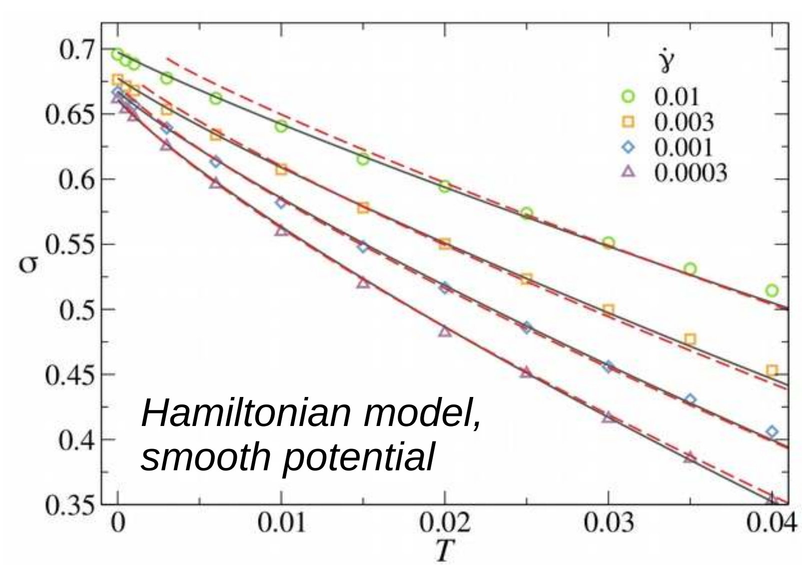
$$\tau_{cT} = \tau_{c0}\left(1 - [AT\ln(\omega_0/C\dot{\gamma})]^{\frac{2}{3}}\right)$$



“J&S’s law” for general α

$$\tau_{cT} = \tau_{c0}\left(1 - [AT\ln(\omega_0/C\dot{\gamma})]^{\frac{2}{3}}\right)$$





If $\quad \dot{\gamma}(\sigma, T) = T^{\psi} G((\sigma - \sigma_c)/T^{1/\alpha})$

and, at least for $\sigma \ll \sigma_c$, we ask

$$G(x) = C_1 \exp(C_0(-x)^{\alpha})$$

to reflect the thermal activation over barriers that scale as

$$(\sigma_c - \sigma)^{\alpha}$$

...we can invert to obtain

$$\sigma(T) = \sigma_c - [C_0^{-1} T \log(C_1 T^{\psi}/\dot{\gamma})]^{1/\alpha}$$

Which can be matched with the J&S expression (up to non-leading terms) if α=3/2

EEF, AB Kolton & EA Jagla
Phys. Rev. Materials **5**, 115602 (2021)

(see also [Dasgupta et al. PRB 87, 020101 (2013)])

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(FULLY-COUPLED) MEAN-FIELD DEPINNING & YIELDING

$$G_{\mathbf{q}} \sim \mathbf{q}^0$$

EEF & EA Jagla, *PRL* **123**, 218002 (2019)

Combined FC-MF-depinning and yielding problems

EEF & EA Jagla, *PRL* **123**, 218002 (2019)

Depinning & yielding common e.o.m. (in Fourier space)

$$\eta \frac{de_{\mathbf{q}}}{dt} = G_{\mathbf{q}} e_{\mathbf{q}} + f_p(e)|_{\mathbf{q}} + \sigma$$

(Fully-connected) mean-field depinning

$$G_{\mathbf{q}}^{\mathtt{MFD}} = -1$$

Yielding in *d=2*

$$G_{\mathbf{q}}^{Y} = -\frac{(q_x^2 - q_y^2)^2}{(q_x^2 + q_y^2)^2} \qquad (\mathbf{q} \neq 0)$$

In both cases $G_{\mathbf{q}} \sim \mathbf{q}^0$

(propagator range is the same, angular dependence changes)

We propose a linear combination of both kernels

$$G_{\mathbf{q}} \equiv (1-\varepsilon) G_{\mathbf{q}}^{\mathtt{Y}} + \varepsilon G_{\mathbf{q}}^{\mathtt{MFD}}$$

Similarly in yielding *d=3*

$$G_{\mathbf{q}}^{3\mathrm{D}} = \frac{2q_x^2(q_y^2 + q_z^2)}{(q_x^2 + q_y^2 + q_x^2)^2} - 1.$$

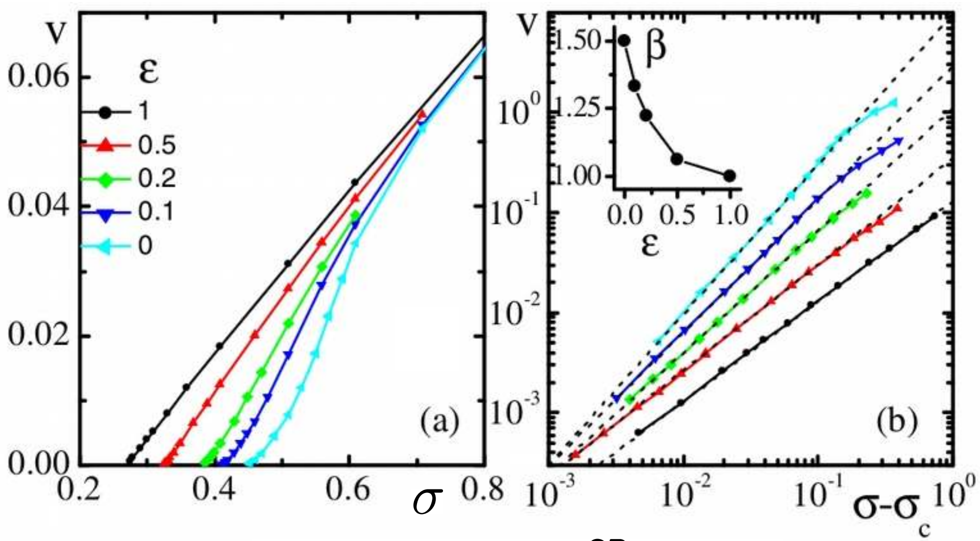
and switch among them with a parameter ε mean-field depinning **(ε=1)** and yielding **(ε=0)**

Smoothly from mean-field depinning to 2D yielding

Flowcurves

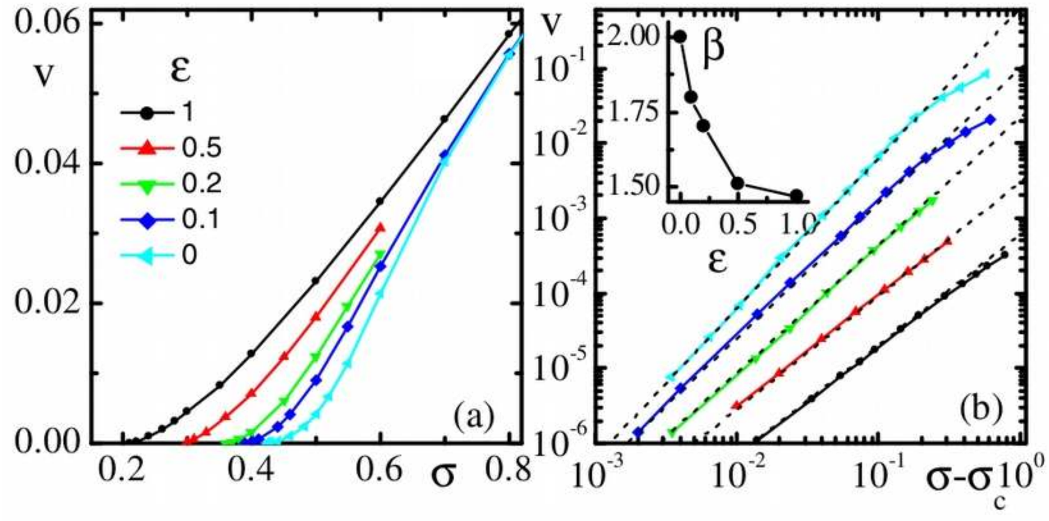
Variation between mean-field depinning **(ε=1)** and yielding **(ε=0)**

For *cuspy* disorder potential



$$\beta_{\mathtt{MFD}} = 1 \rightarrow \beta^{\mathrm{2D}}_{\mathtt{Y}} = 1.5$$

For *smooth* disorder potential



$$\beta_{\mathtt{MFD}} = 1 \rightarrow \beta^{\mathrm{2D}}_{\mathtt{Y}} = 2$$

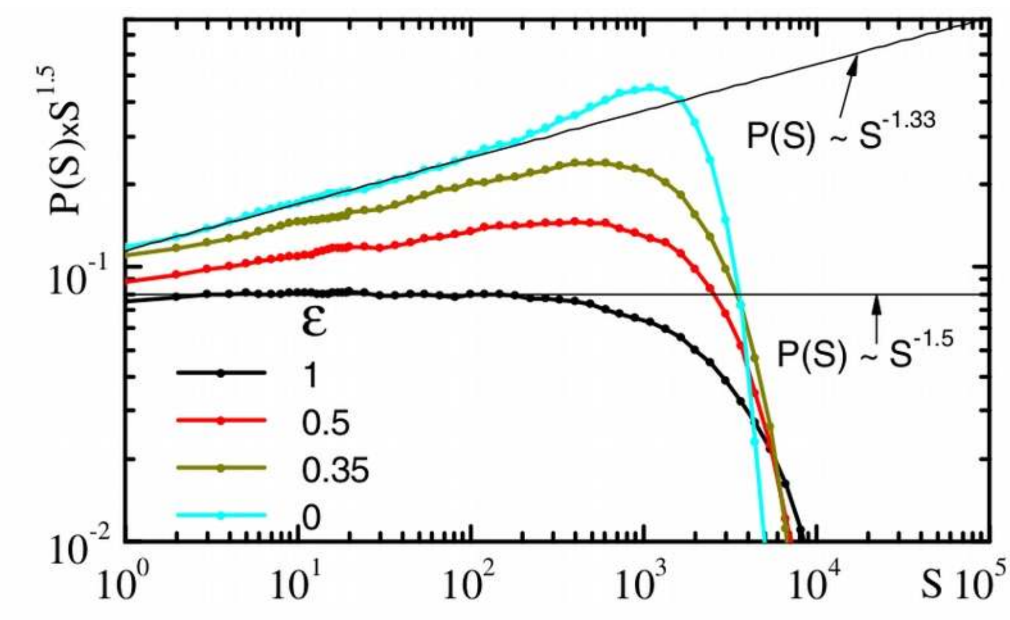
Smooth change of exponents!

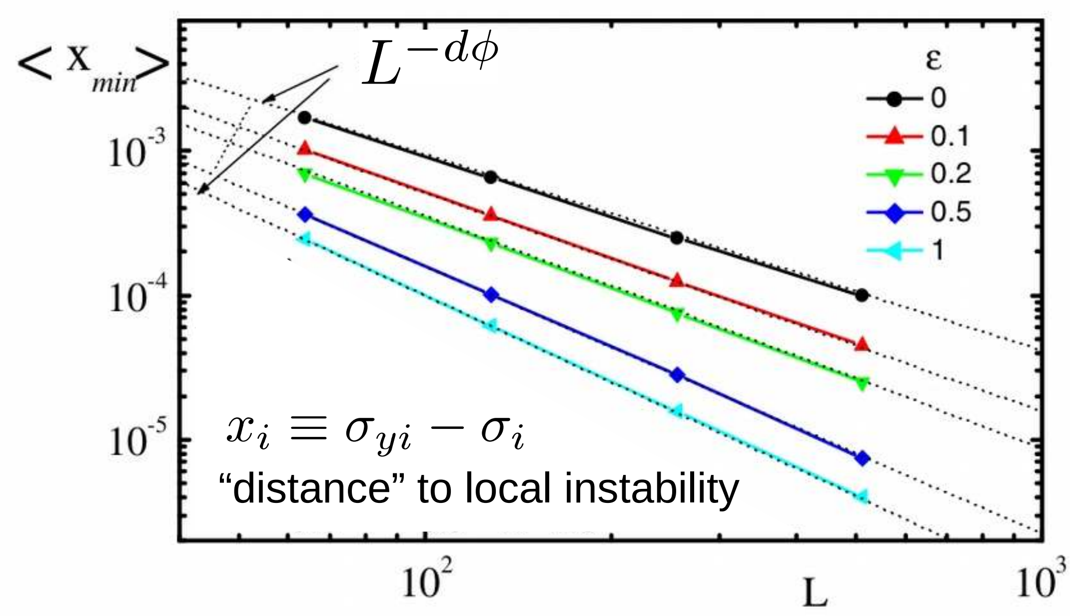
Why is this surprising? If we combine different criticalities, mixing two kernels **with different ranges**, the system will ultimately **display the critical exponents** corresponding to the **longest range interactions**. Here, instead, FC-MFD and yielding perfectly **coexist**.

EEF & EA Jagla, *PRL* **123**, 218002 (2019)

Smoothly from mean-field depinning to 2D yielding

Avalanches and x_{min} scaling





$$P(S) \sim S^{-\tau} f(S/S_{\mathtt{cut}})$$

$$\tau_{\mathtt{MFD}} = 3/2$$

$$\tau_{\mathtt{2DY}} \simeq 1.33$$ (quasistatic protocol)

$$\langle x_{\mathtt{min}} \rangle \sim N^{-\phi} \sim L^{-d\phi}$$

$$\phi_{\mathtt{MFD}} = 1$$

$$\phi_{\mathtt{2DY}} \simeq 2/3$$

*Message: both problems are **contained** in a **general** one*

EEF & EA Jagla, *PRL* **123**, 218002 (2019)

3rd observation:

Eshelby's propagator is $\sim q^0$, just as fully-connected mean-field depinning.

Smooth variation of exponents when switching between MF depinning and 2D yielding propagators.

Proposal:

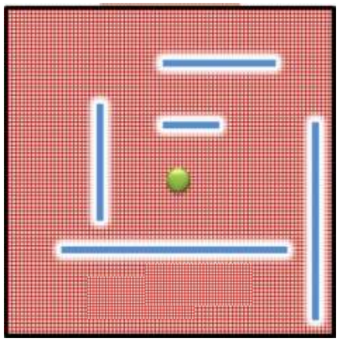
Yielding in finite dimensions should support a mean-field or “one particle” description

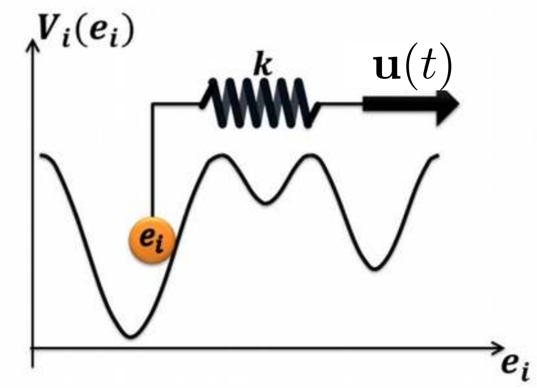
Proposal:

Yielding in finite dimensions should support a mean-field or "one particle" description.

➔ *Plug-in the effective noise in a one particle model and recover the flowcurves.*

EFFECTIVE NOISE & ONE PARTICLE DESCRIPTION





EEF & EA Jagla *Soft Matter* **15**, 9041 (2019)

EEF, AB Kolton & EA Jagla Phys. Rev. Materials **5**, 115602 (2021)

Mechanical noise **measured** at a generic point

d=2

T=0

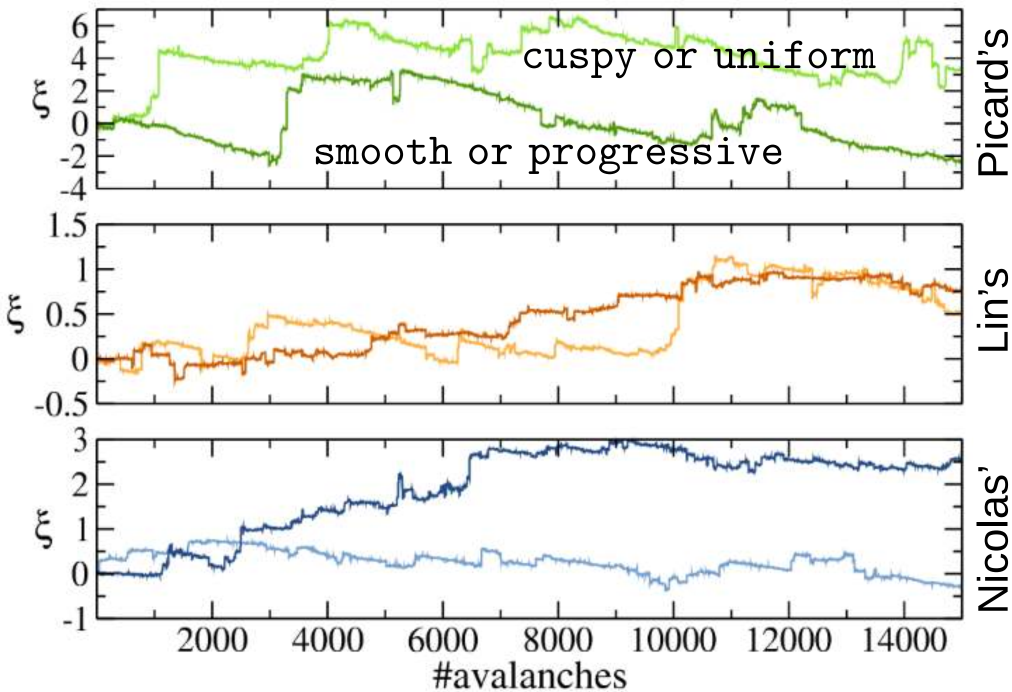
What if we wanted to describe our dynamics with an **effective mean-field** ?

$$\partial_t \sigma_i = \mu\dot{\gamma} - g_0 n_i \frac{\sigma_i}{\tau} + \sum_{j\neq i} G_{ij} n_j \frac{\sigma_j}{\tau}$$

$$\partial_t \sigma(t) = \mu\dot{\gamma}^{\mathtt{ext}} - g_0 n(t) \frac{\sigma(t)}{\tau} \boxed{+\,\xi}$$

noise

In the **quasistatic** limit:



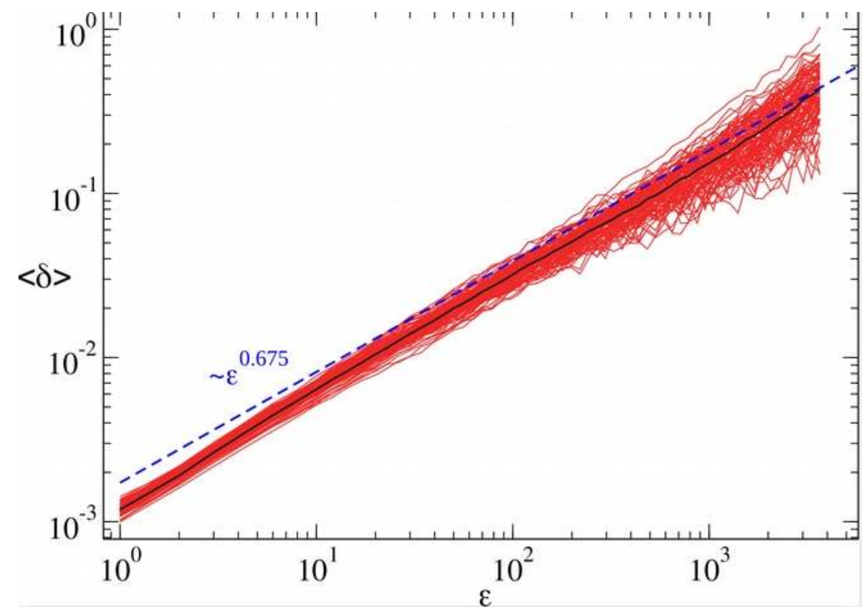
Accumulated noise at a point (during an avalanche)

$$\xi_i = \sum_{\mathtt{ava\#}} \delta\xi_i$$

Assuming the noise signal has a Hurst exponent H

$$\xi(kx) = k^H \xi(x)$$

Run a DFA (De-trended Fluctuation Analysis):



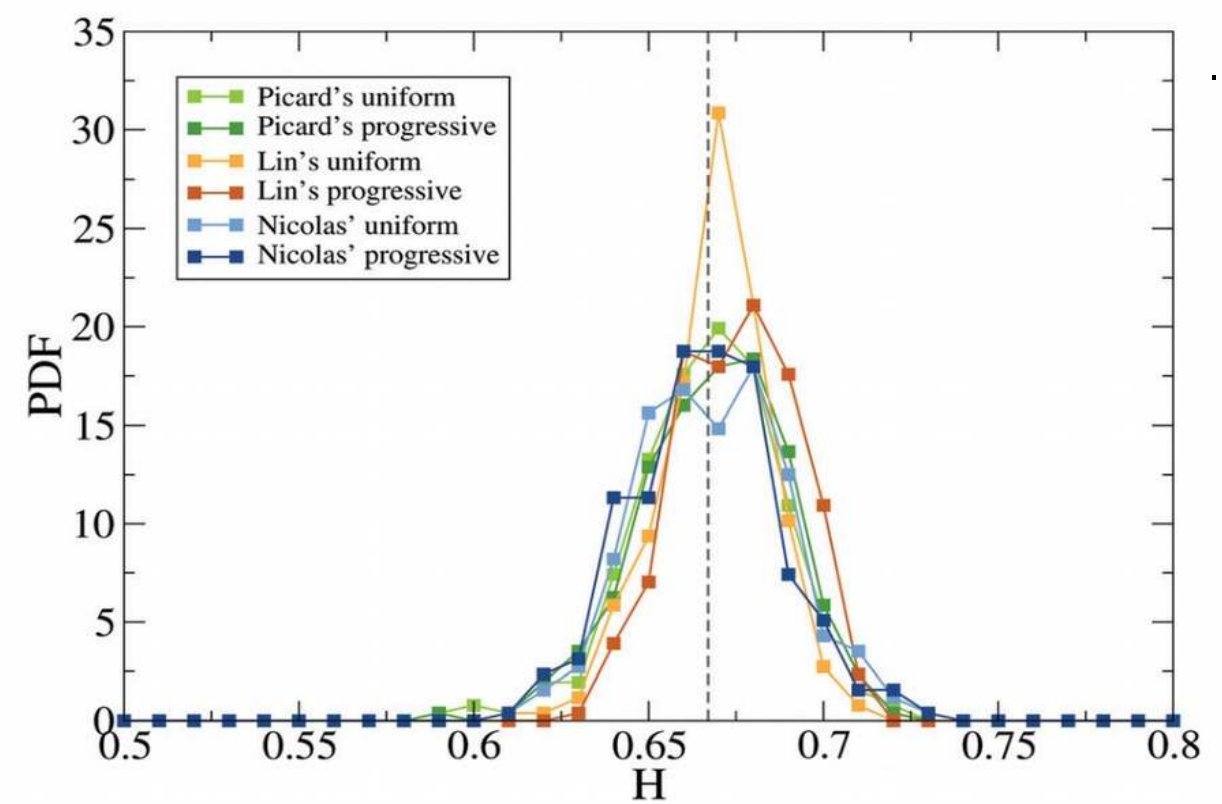
We do statistics of the observed heights δ of boxes of length ε along the noise signal (suppressing a global trend)

Mechanical noise **Hurst exponent**

d=2 *T=0*

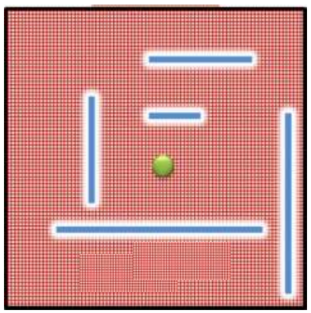
$$\partial_t \sigma(t) = \mu\dot{\gamma}^{\mathtt{ext}} - g_0 n(t)\frac{\sigma(t)}{\tau} \boxed{+\,\xi}$$

$$\xi(kx) = k^H \xi(x) \quad ?$$



...via DFA

Instead of coming from single-site kicks, the noise is produced by avalanches



$H \simeq 0.67$ is found for 6 different EP models

A Random Walk has *H=0.5*

A noise generated by single Eshelby's has *H=1*

$$P(\delta\xi) \sim \frac{1}{|\delta\xi|^{1+\frac{1}{H}}} \qquad H = 1$$

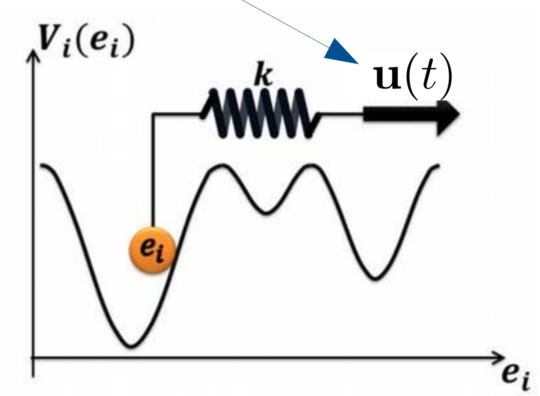
[Lin&Wyart, PRE **97**, 012603 (2018)]

Stochastic “Prandtl-Tomlinson” model

One particle in a quenched potential V(x) with stochastic driving and thermal noise

$$\frac{dx}{dt} = -\frac{dV}{dx} + k_0\,(u(t) - x)\ \ +\sqrt{T}\eta_0(t)$$

$$\frac{du}{dt} = b\dot{\gamma} + a\dot{\gamma}^H \eta_H(t) \qquad\qquad P(\eta_H) \sim \frac{1}{|\eta_H|^{\frac{1}{H}+1}}$$



The stress is estimated as $\sigma \equiv \overline{k_0(u(t) - x(t))}$

At T=0 “analytically” can be found

$$\sigma - \sigma_c \sim \dot{\gamma}^{\frac{\omega H}{\omega H - H + \omega}}$$

In other words, $\boxed{\beta = \frac{1}{H} + 1 - \frac{1}{\omega}}$

ω is the behavior of the pinning force around the transition points $-dV(x)/dx \simeq Ax^{\omega}$

ω=1 for *cuspy* potential, *ω=2* for *smooth* potential

At T>0

$$\sigma - \sigma_c = \dot{\gamma}^{\frac{\omega H}{\omega H + \omega - H}} f\left(\frac{\dot{\gamma}^{\frac{(\omega+1)H}{\omega H + \omega - H}}}{T}\right)$$

Which can be worked into the more standard form

$$\dot{\gamma} = T^{\psi} G\left((\sigma - \sigma_c)/T^{1/\alpha}\right)$$

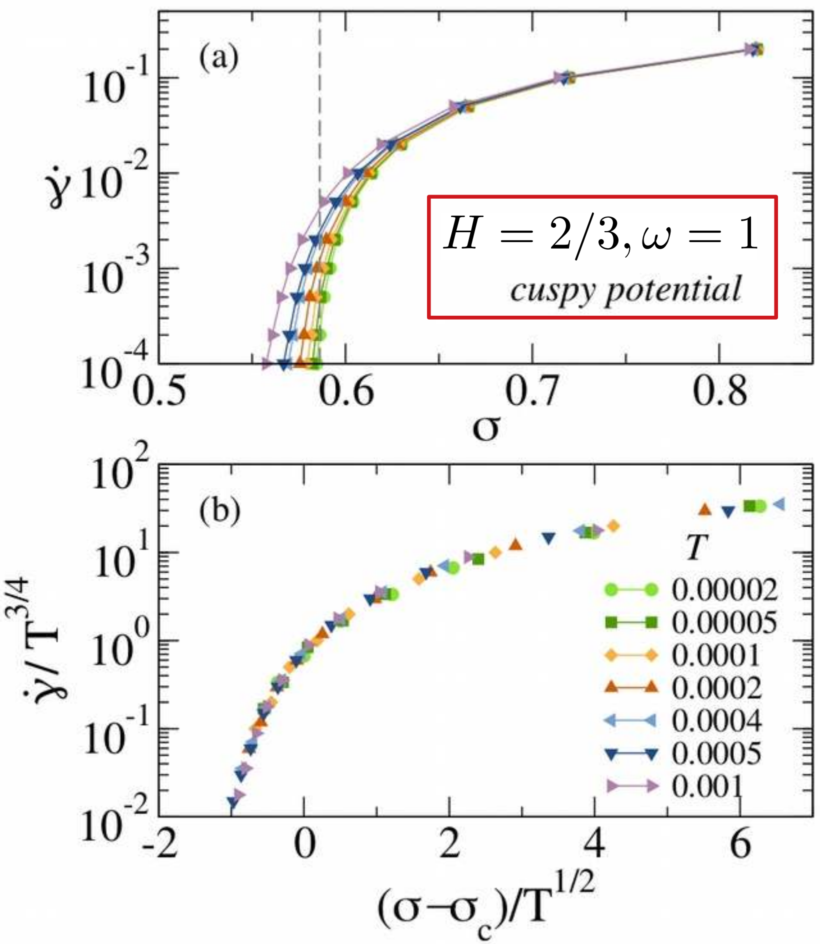
with $\boxed{\psi = \frac{\omega H + \omega - H}{(\omega + 1)H}}$ $\boxed{\alpha = 1 + \frac{1}{\omega}}$

$$\boxed{\beta = \psi\alpha = \frac{1}{H} + 1 - \frac{1}{\omega}}$$

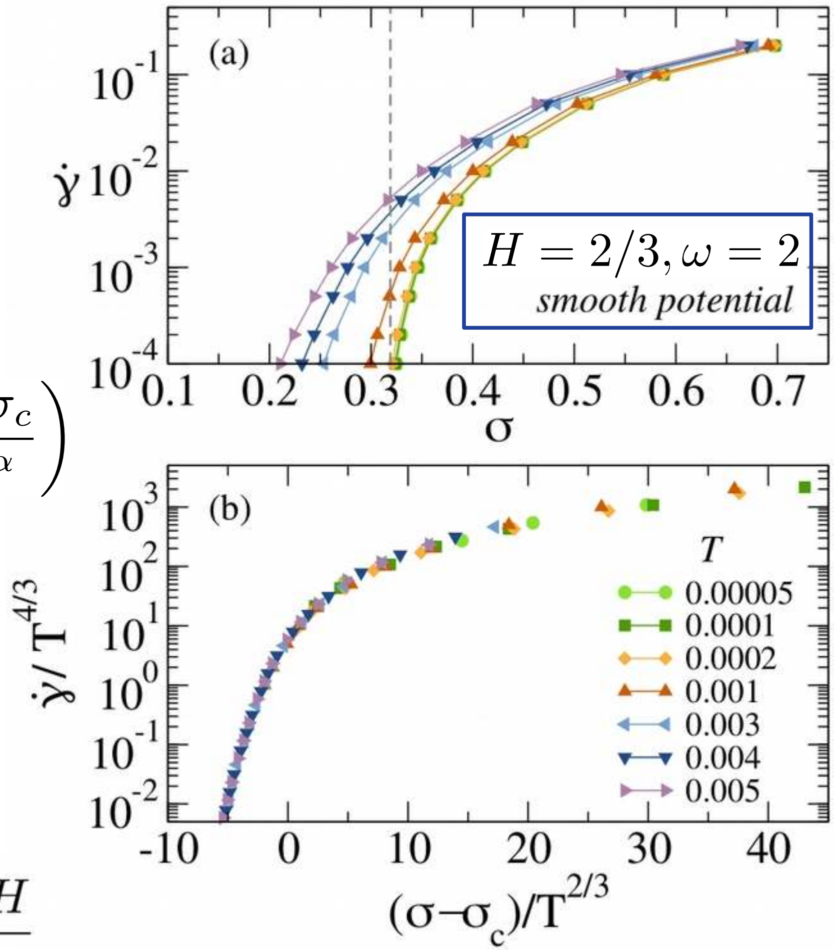
EEF, AB Kolton & EA Jagla Phys. Rev. Materials **5**, 115602 (2021)

Thermal rounding: “one-particle” PT model

“cuspy”



“smooth”



$$\frac{\dot{\gamma}(\sigma, T)}{T^{\psi}} = \mathcal{G}\left(\frac{\sigma - \sigma_c}{T^{1/\alpha}}\right)$$

$$\alpha = 1 + \frac{1}{\omega}$$

$$\psi = \frac{\omega H + \omega - H}{(\omega + 1)H}$$

$$\beta = \frac{1}{H} + 1 - \frac{1}{\omega}$$

$\beta = 3/2, \alpha = 2, \psi = 3/4$

$\beta = 2, \alpha = 3/2, \psi = 4/3$

EEF, AB Kolton & EA Jagla, Phys. Rev. Materials **5**, 115602 (2021)

Take home messages

- **Equivalence** can be built between **elasto-plastic models** and **manifolds on quenched disorder** approaches.

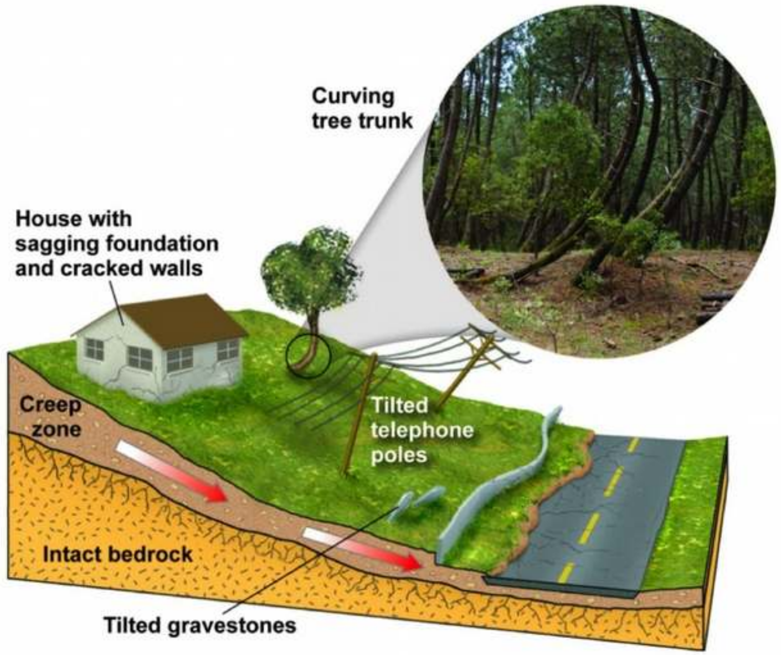
- **Two coexisting universality classes** exist: **“cuspy”** and **“smooth”** potentials (or uniform and progressive yielding rules) for yielding, with **different flowcurve exponent β** (and also z differs), but **“the same”** ***static*** critical exponents: **τ, d_f, ϕ, H**.

- **Thermal rounding** anzats **works without corrections** in yielding at finite dimension.

- **Smooth variation** of exponents when interpolating between **fully-connected** mean-field **depinning** and **yielding** in **finite dimension**.

- ***Yielding in finite dimensions*** can be described as an ***effective ‘mean-field’***, after characterization of the **mechanical noise**.

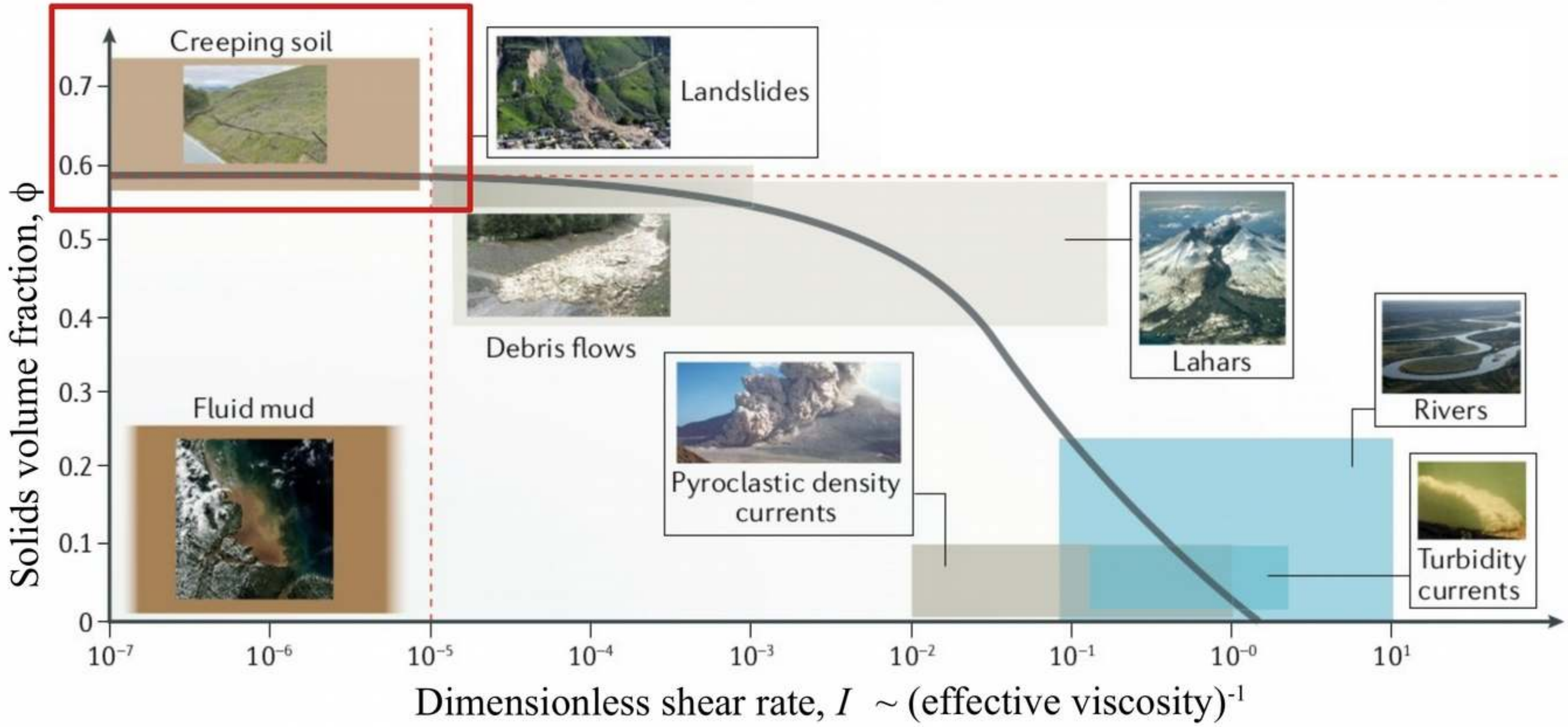
ATHERMAL CREEP BY CYCLIC PERTURBATIONS



EEF & EA Jagla, arXiv:2501.07782

Creeping hill-slopes in Soft Earth Geophysics

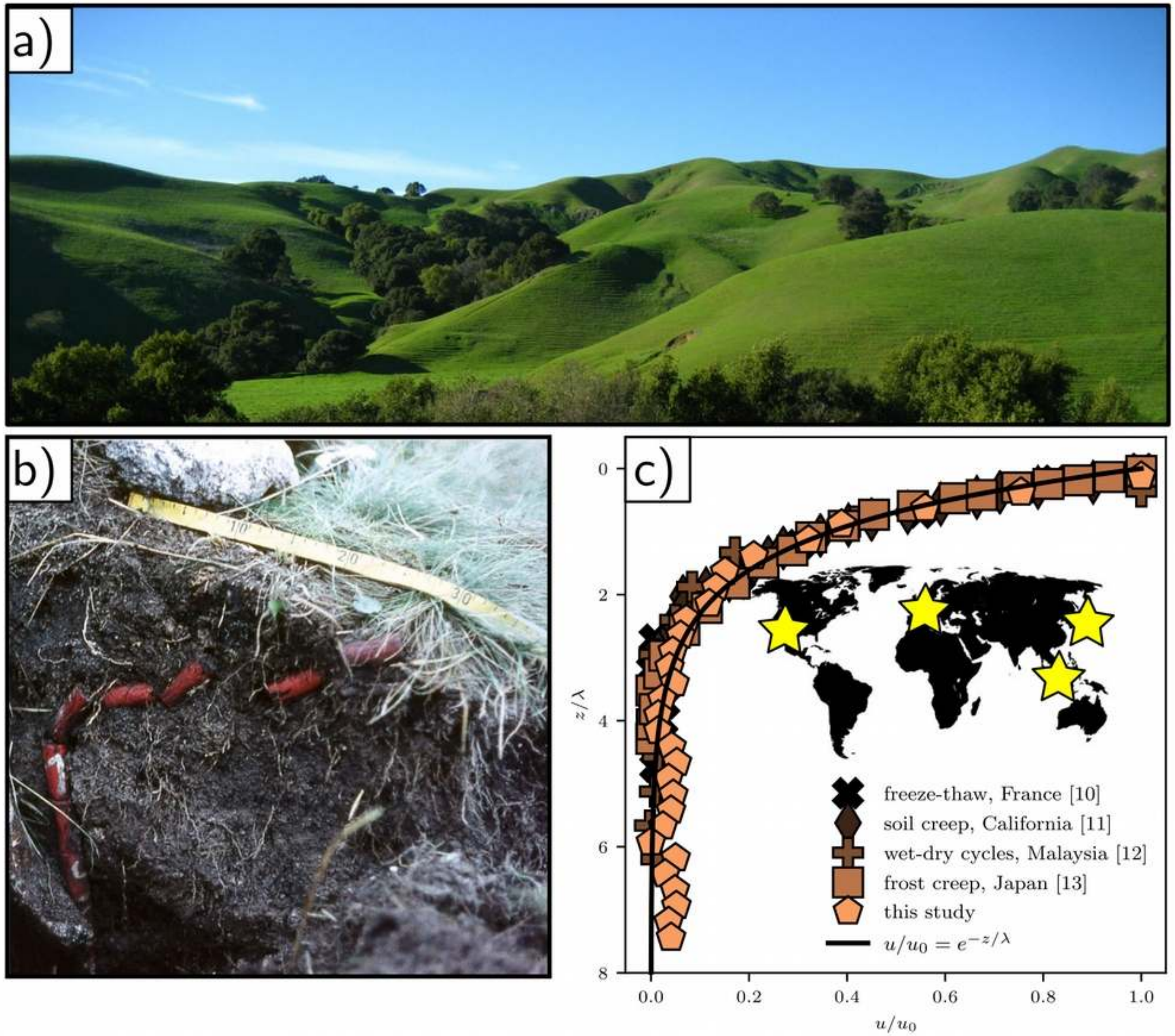
Soft Earth Geophysics



K. E. Daniels and D. J. Jerolmack,
"Viewing earth's surface as a soft-matter landscape",
Nature Reviews Physics 1, 700 (2019).

Creeping hillslopes in Soft Earth Geophysics

Canonical soil-mantled hillslopes, California



Displacement of tracer pegs over a 17-year interval (Poland)

Compilation of soil deformation data

N. S. Deshpande, D. J. Furbish, P. E. Arratia, and D. J. Jerolmack, *The perpetual fragility of creeping hillslopes,* Nature Communications 12, 3909 (2021).

Theoreticians can do experiments too!

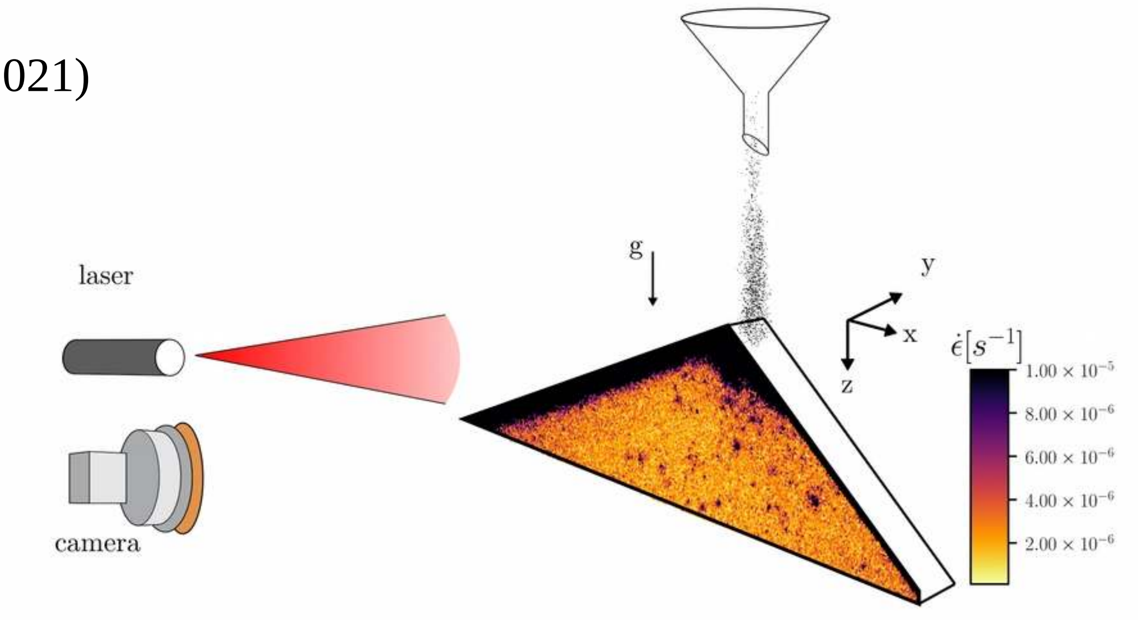
Creeping granular heap

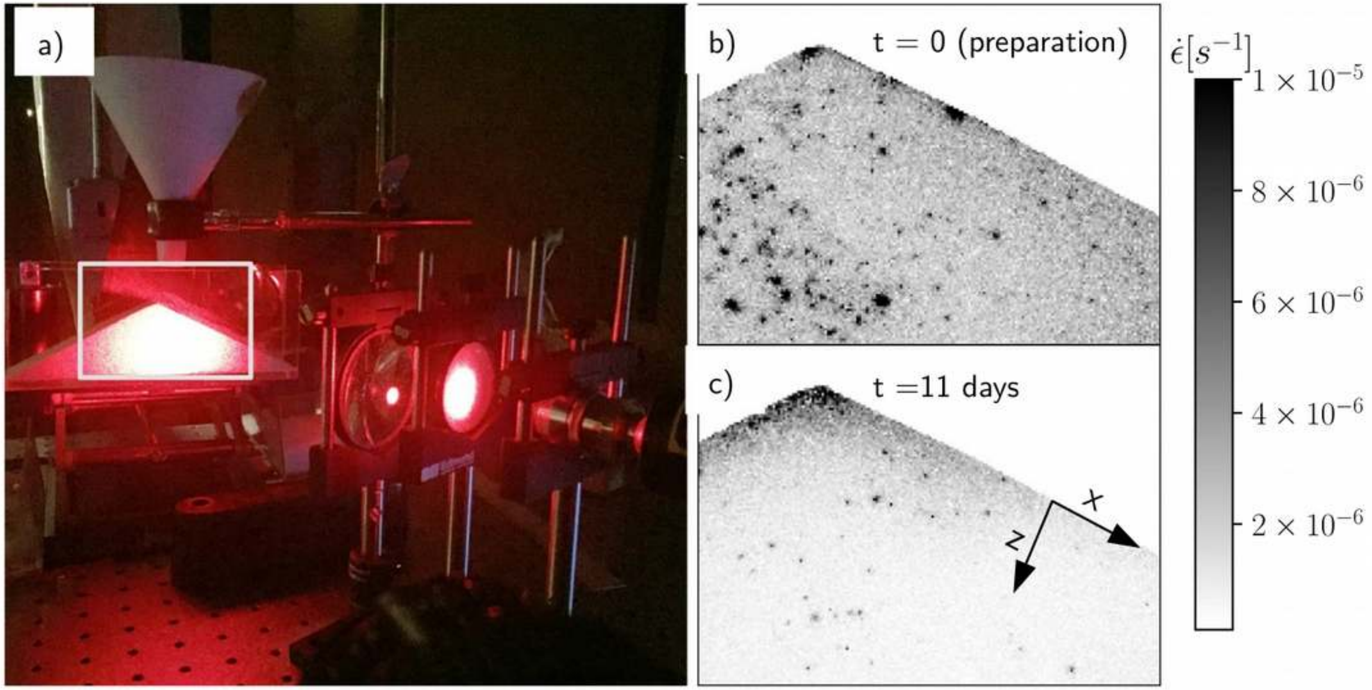
Deshpande et al. Nat. Comm. **12**, 3909 (2021)

Objective: demonstrate the existence of creep in a minimally disturbed *model hillslope*.

Expect *very slow creep rates* (≤10^{-6}m/s)

Measure grain motions via spatially-resolved Diffusing Wave Spectroscopy



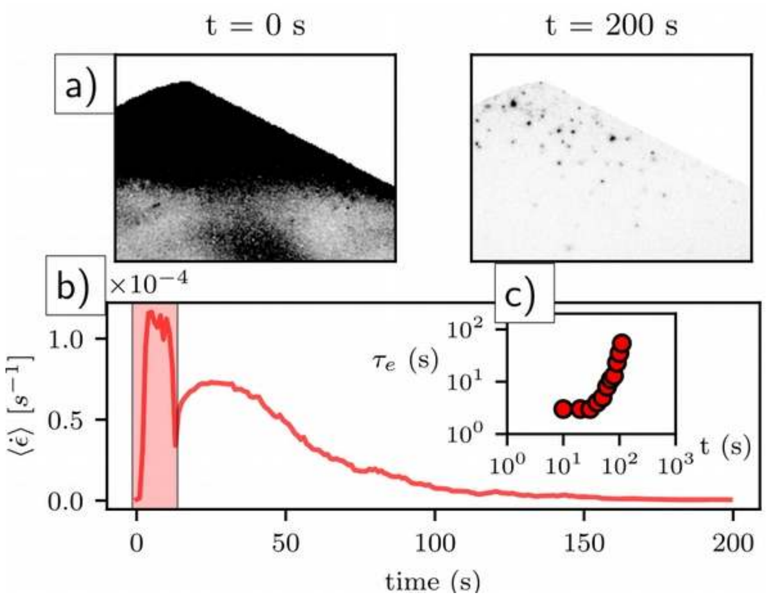


Strain-rate maps

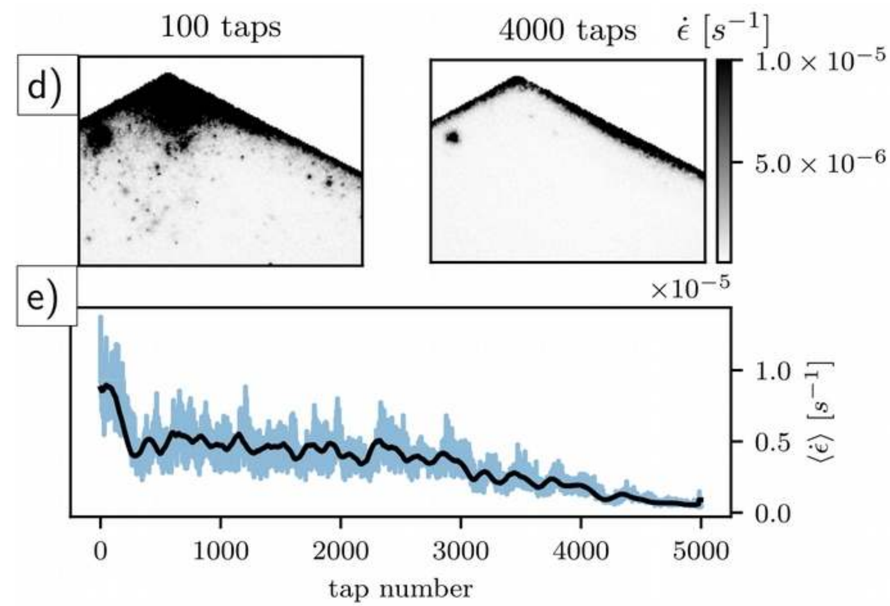
11 days after particle activity persists !

Different experiments testing “geophysically-relevant disturbances”:

Heat pulse



Tapping

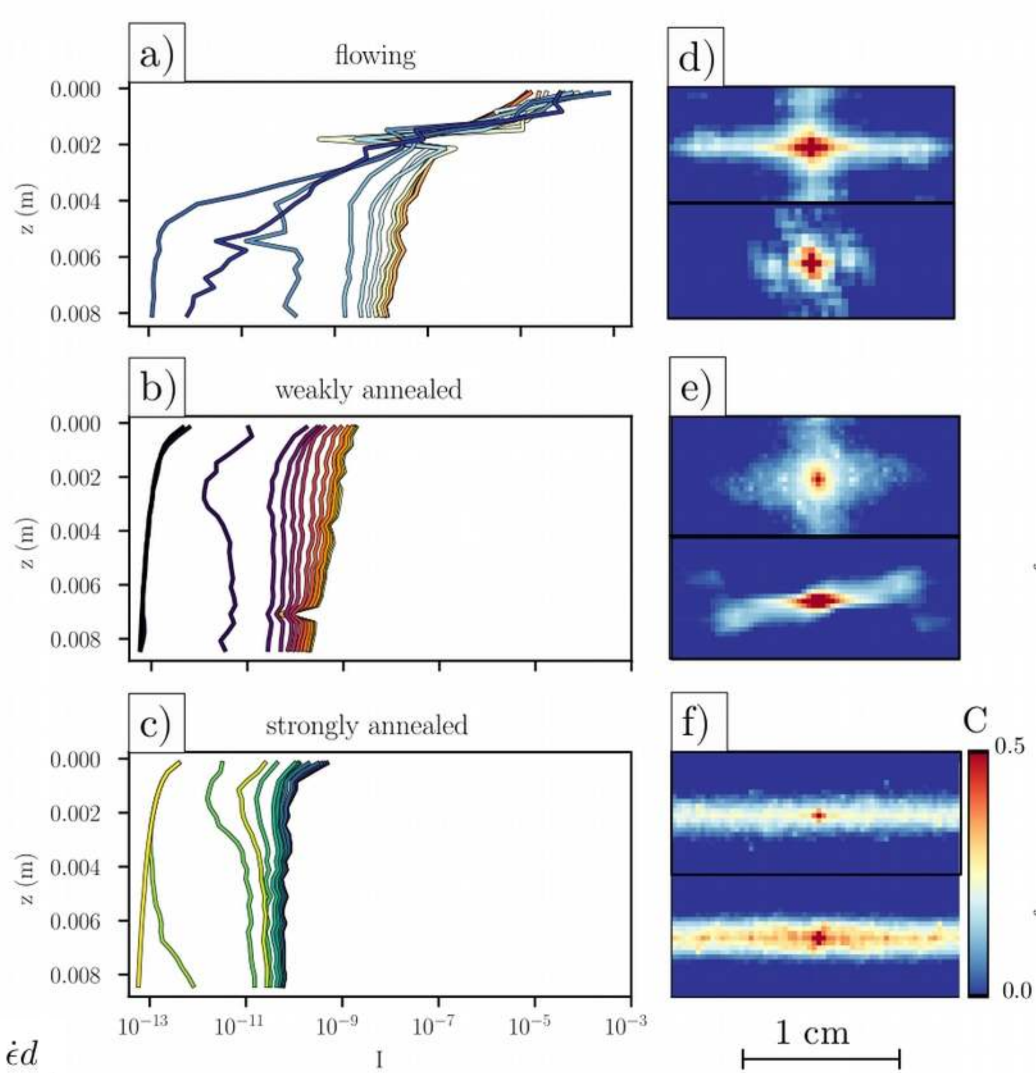


“Creep **occurred for all experiments** and granular materials, and it **persisted over all observed timescales ($1 - 10^6$ s)**”

“By probing a **seemingly static** sandpile with speckle imaging, our experiments have revealed a seething and **ceaseless creeping motion**—even in the near **absence** of mechanical disturbances”

“Our system is **intentionally prepared close to the critical state**. Certainly, this means that **creep rates are nearly as fast as they can be**, and we expect them to slow exponentially with decreasing slope.”

N. S. Deshpande, P. E. Arratia, and D. J. Jerolmack, *“Athermal granular creep in a quenched sandpile”*, arXiv:2402.10338 (2024)



$$I = \frac{\dot{\epsilon} d}{\sqrt{\frac{P}{\rho}}}$$

C: spatial correlations of the strain field

Flowing layer

Within the surface flowing layer the dimensionless strain rate diminishes with depth, there is an absence of spatial correlations, and there is no aging dynamics.

Bulk

Beneath this layer, the bulk creeps via localized avalanches of plasticity, and there is significant aging.

“Surprisingly, at the cessation of surface flow and the ‘quenching’ of the pile, creep persists in the absence of the flowing layer; albeit with significant differences for a pile that experiences a long duration of surface flow (strongly annealed) and one where flow during preparation does not last long (weakly annealed).”

Which are the creep mechanisms affecting quiescent heaps / soil dynamics?

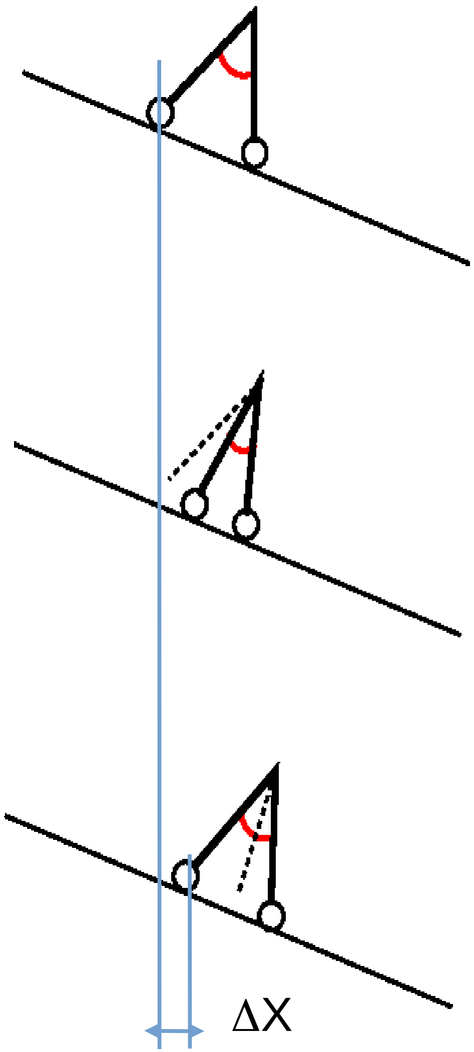
- Certainly not thermally activated creep (granular systems are ‘athermal’)

- Mechanically induced creep (wind, water, animals, earthquakes)

  Yes, a possible mechanism, which explains creep in part

-Creep facilitated by periodic variation of parameters
(day/night, winter/summer temperature/humidity variations)

A very simple idea, but somehow under exploited



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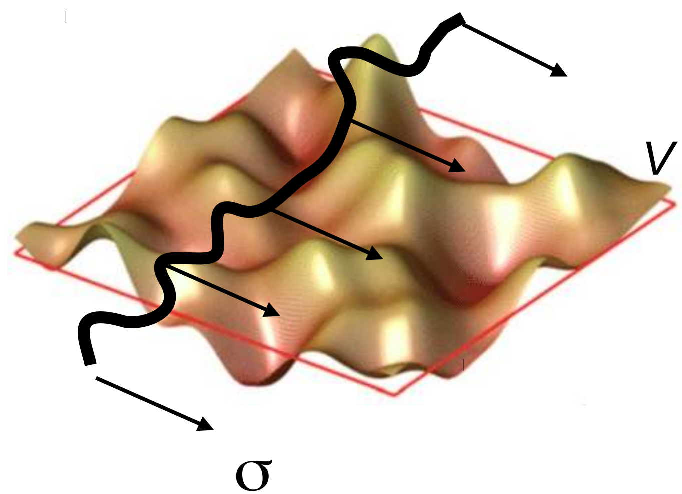
Proceedings of the Royal Society of London 7, 333 (1856).

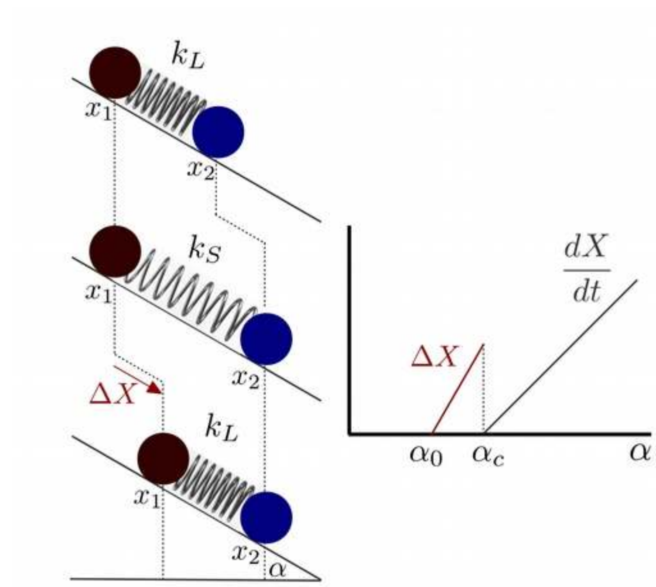
J. G. Croll, “Thermally induced pulsatile motion of solids”,
Proceedings of the Royal Society A 465, 791 (2009).

B. Blanc, L. A. Pugnaloni, and J.-C. Géminard,
“Creep motion of a model frictional system”, Phys. Rev. E 84, 061303 (2011).

“Thermo-mechanical ratcheting” in mechanical engineering

Our approach: models of depinning and yielding





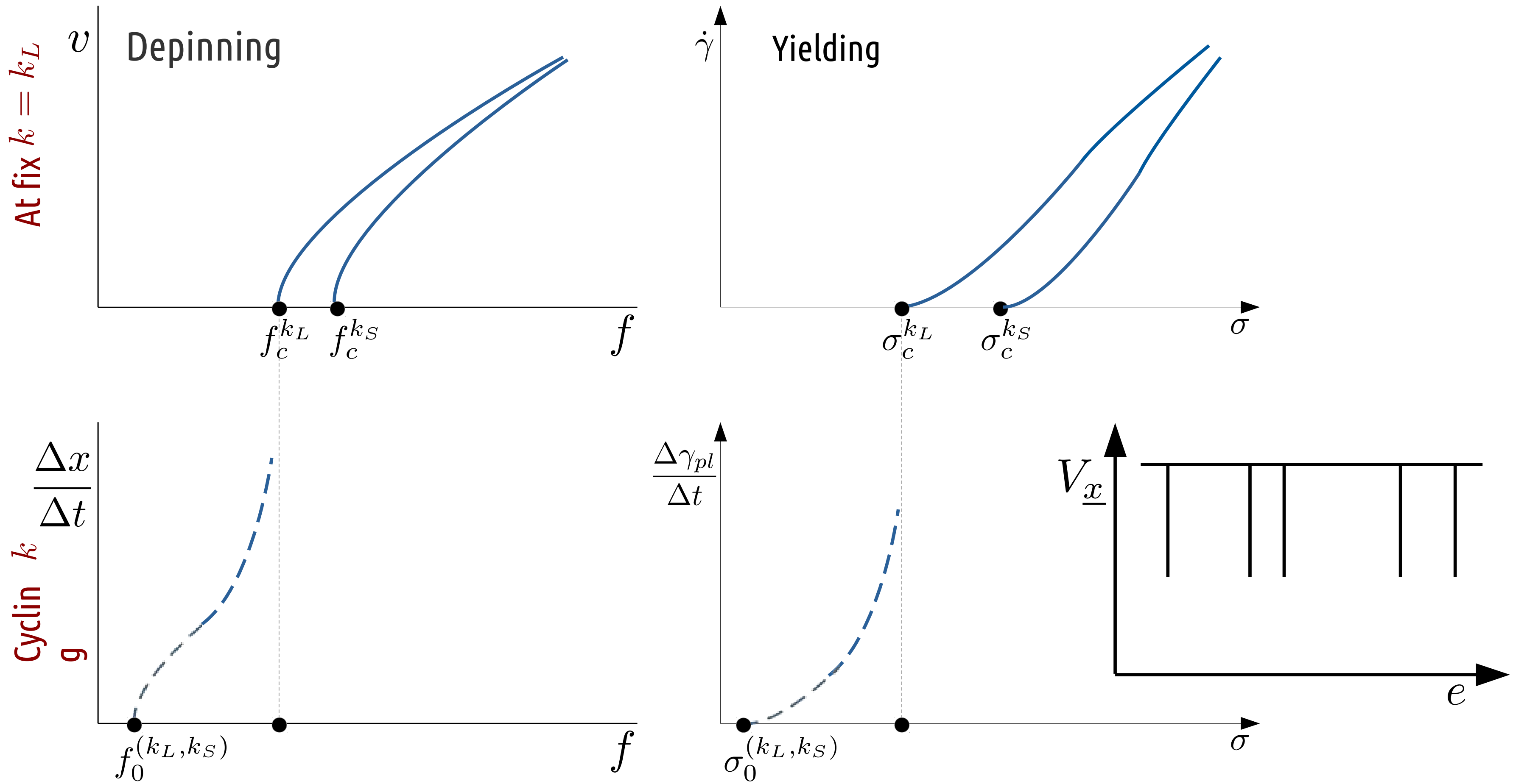
Cyclic Perturbations Facilitate Athermal Creep in Yield-Stress Materials

EEF and EA Jagla, arXiv:2501.07782

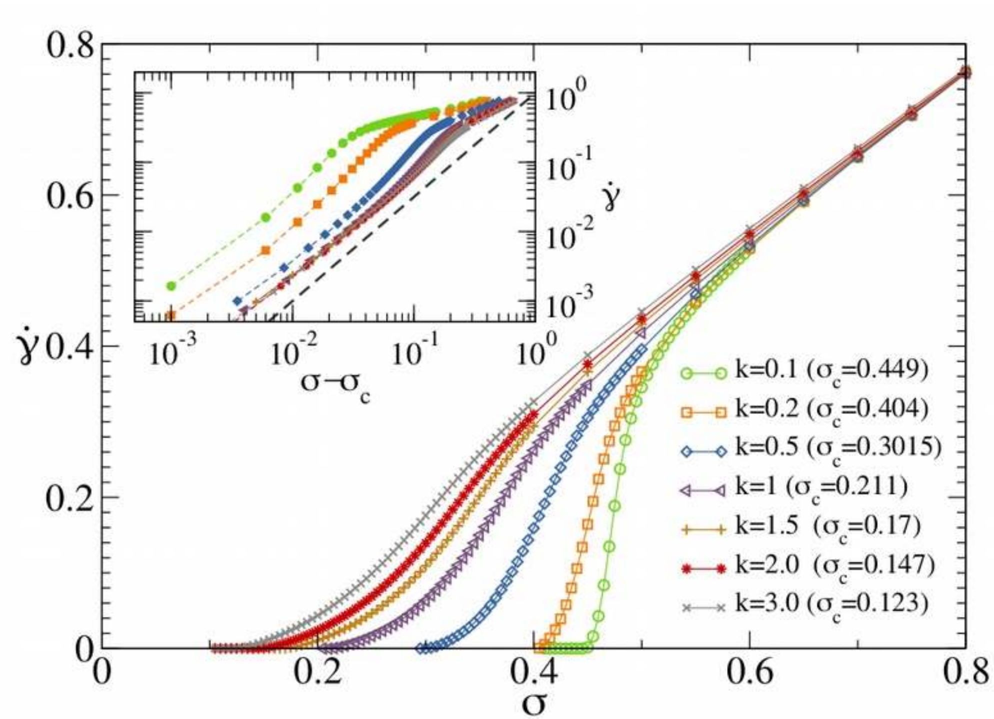
$$G_{\mathbf{q}} = k\mathbf{q}^2 \longleftarrow \boxed{G_{ij} \to G_{\mathbf{q}}} \longrightarrow G_{\mathbf{q}} = k\frac{(q_x^2 - q_y^2)^2}{(q_x^2 + q_y^2)^2}$$

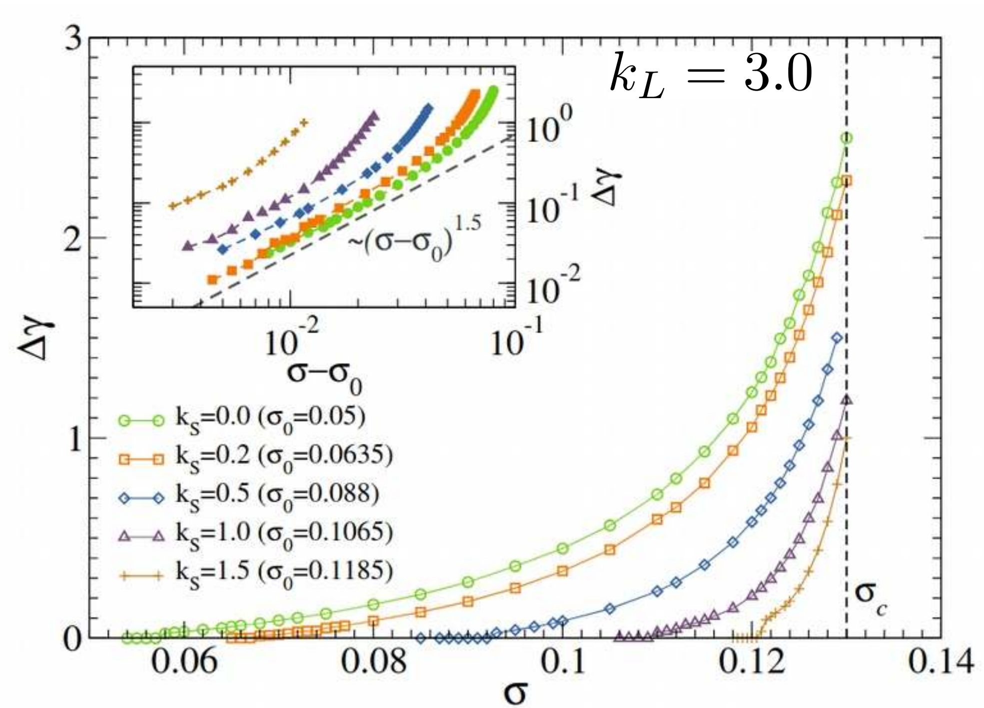
n.n. interactions (d=2)

Eshelby l.r. interactions (d=2)



Yielding flowcurves for different *k* and $\Delta\gamma$ advance under $k=k_L \leftrightarrow k=k_S$ oscillations





$$\dot{\gamma} \sim (\sigma - \sigma_c)^{\beta_y}$$

$$\beta_y = 3/2$$

Under very slow oscillations of ***k*** we reach a steady state of fix advance $\Delta\gamma$ *per period*

Deformation $\Delta\gamma$ is maximal closer to σ_c

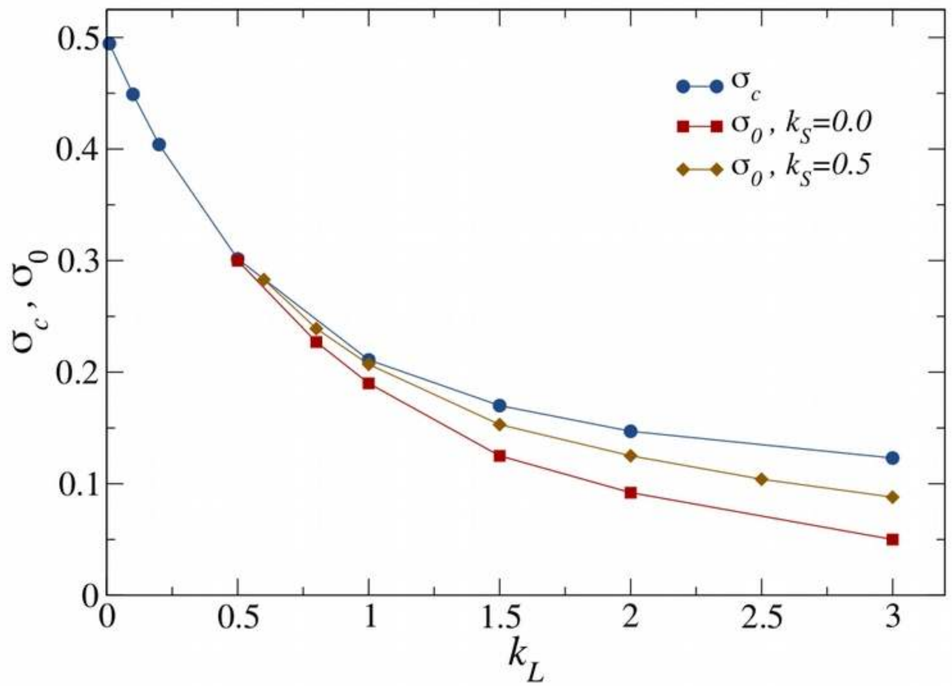
Criticality at σ_c moves now to σ_0 !!

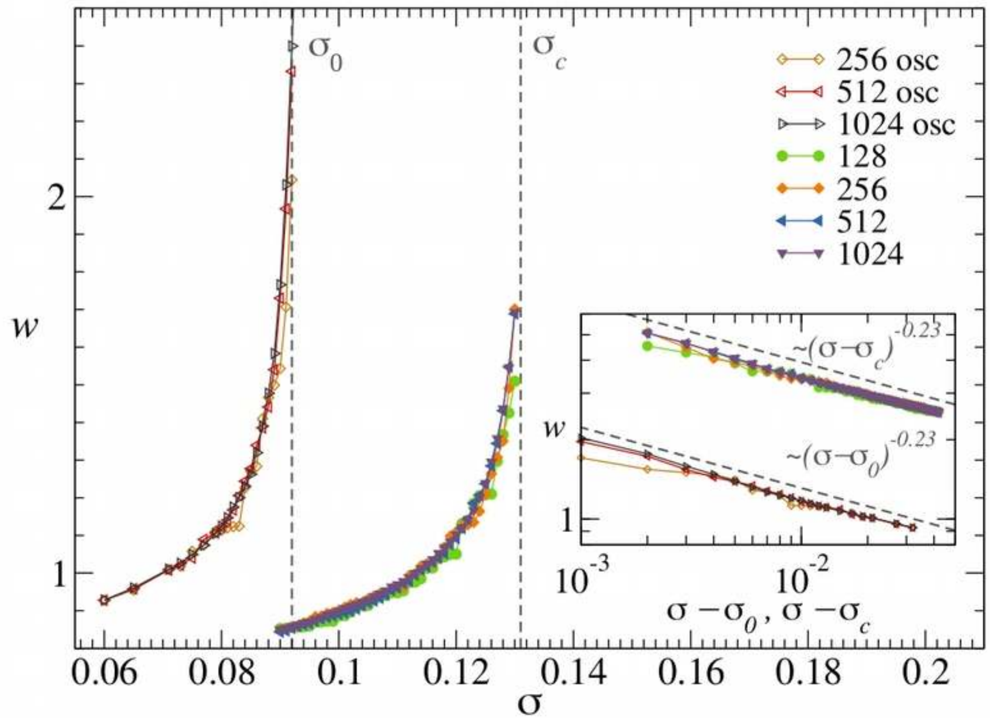
apparent same exponents observed

$$\Delta\gamma \sim (\sigma - \sigma_c)^{1.5}$$

Range of Δγ>0 and divergent manifold width

$$w^2 = \overline{x_i^2} - \overline{x_i}^2$$





Range in which we observe subcritical flow

$$\eta \equiv (\sigma_c - \sigma_0)/\sigma_c$$

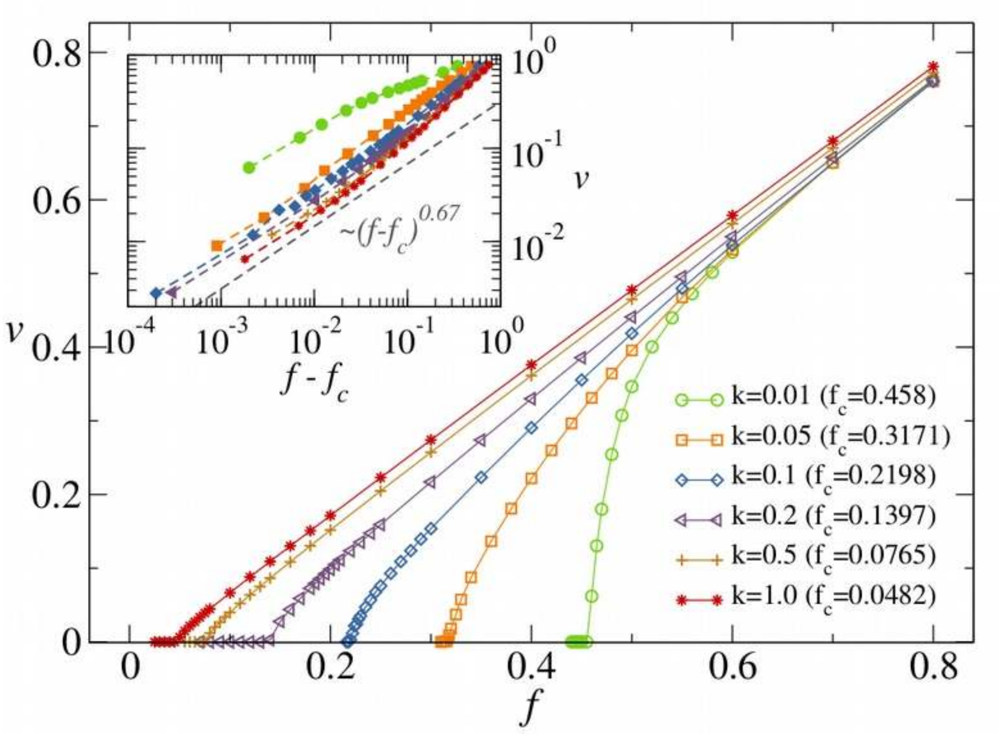
maximal at k_S=0

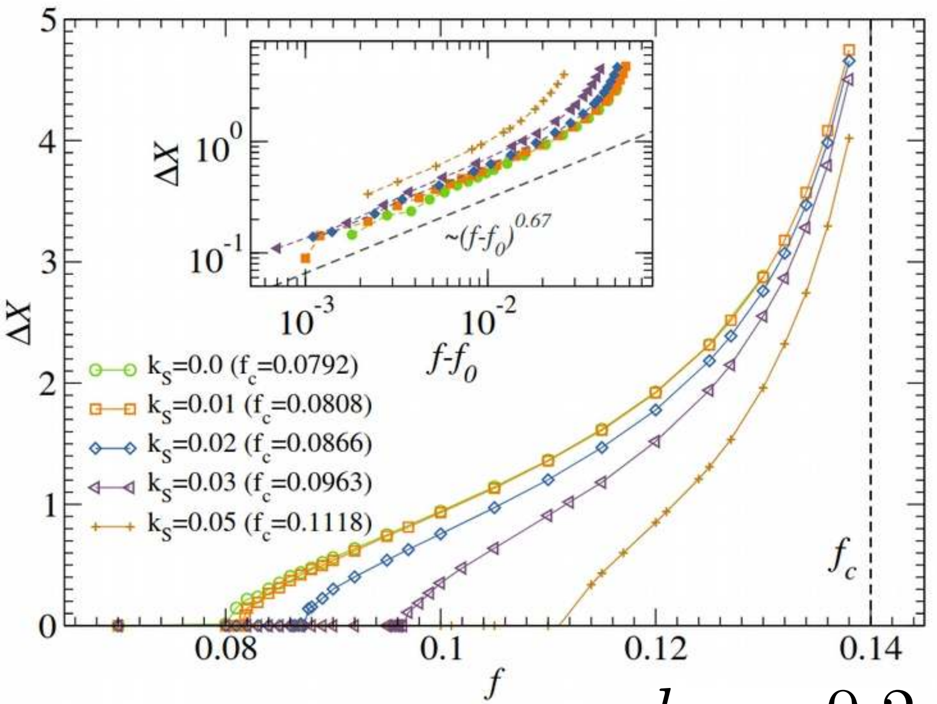
For a fix k_S, η increases for larger k_L

Width *w* 'diverges' with the same exponent:
- in σ_c for the simulation at fixed $k=k_L$
- in σ_0 for the simulation at oscillating $k_L \leftrightarrow k_S$

More evidence for criticality moving from σ_c to σ_0 when oscillating *k*

Depinning characterisics for different k and ΔX advance under $k=k_L \leftrightarrow k=k_S$ oscillations





$$k_L = 0.2$$

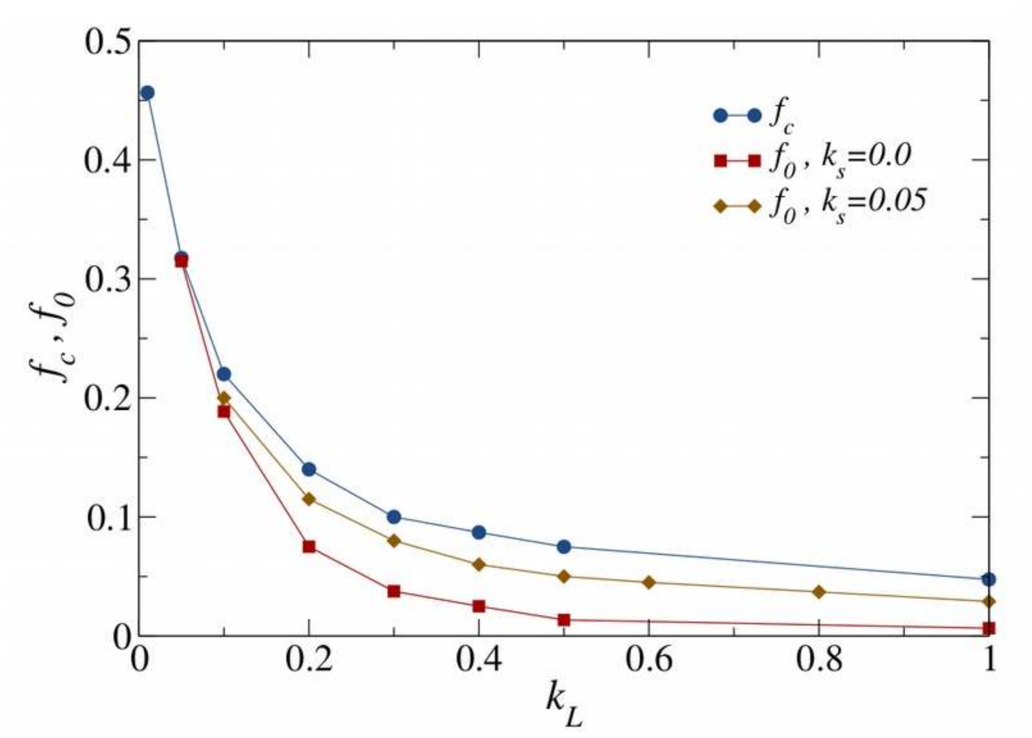
$$v \sim (f - f_c)^{\beta_d}$$

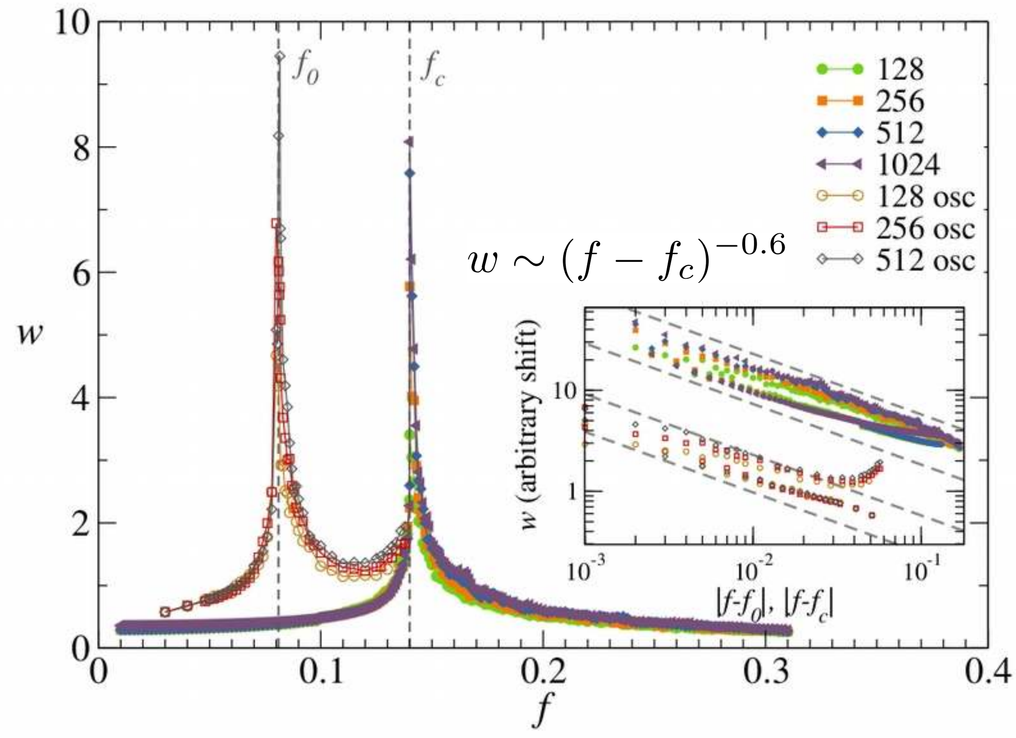
$$\beta_d \simeq 0.67$$

$$\Delta X \sim (f - f_0)^{0.67}$$

Criticality at f_c moves now to f_0 !!

Range of ΔX>0 and divergent interface width





$$w \sim \xi^{\zeta} \qquad \xi \sim (f - f_c)^{-\nu}$$

$$w \sim (f - f_c)^{-\nu\zeta}$$

For 2D short-range depinning

$$\zeta = 0.75 \quad \nu = 0.8 \quad w \sim (f - f_c)^{-0.6}$$

Schematic movement of the oscillatory athermal creep

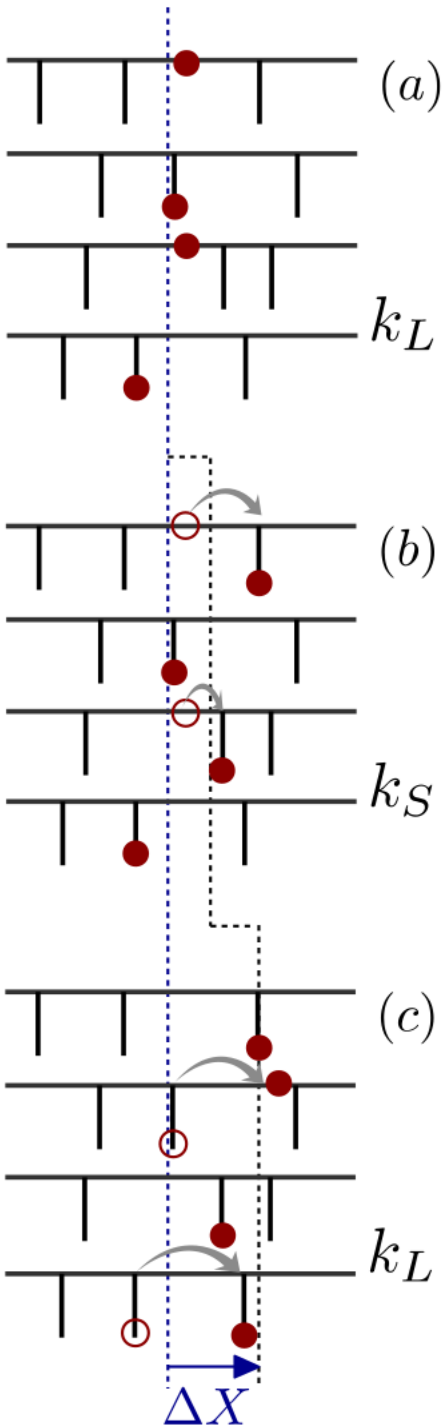


FIG. 11. Qualitative evolution of the configuration of the system when k passes from a high value k_L (a) to a small one k_S (b) and increases again to k_L (c). The same four spatial positions i are depicted in each case. A constant stress f (pointing to the right) is present in all cases. Although the parameters of the system in (a) and (c) are identical, the system configuration is not, the mean position moved to the right from (a) to (c).

Summary

-When the external driving force f is below the critical threshold f_c required for a steady deformation, there is a regime in which the system exhibits synchronized evolution with the periodic variation of k (the global elastic rigidity).

-The deformation per cycle, ΔX, decreases as f is reduced and vanishes at a well-defined threshold f_0, very much like "endurance limit" of fatigue.

-The system likely exhibits criticality at f_0, analogous to its critical behavior at f_c.

-Creep is more notorious close to σ_c.

-Granular experiments showing "ceaseless creeping motion" support indirectly our hypothesis: periodic environmental changes (temperature/humidity) induce periodic oscillations in the system's effective internal parameters.

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