Supplementary Material for
"Elastic interfaces on disordered substrates: From mean-field depinning to yielding"

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Technical description of the stress-controlled and quasistatic strain-controlled protocols used to determine the flowcurves and the avalanche statistics.

A. Simulation protocols

We do straightforward simulations of two-dimensional systems described by Eq.(1) integrated with a first order Euler method. In each integration step, the term $\sum_j G_{ij} e_j$ is treated in Fourier space, computed as $\sum_{\mathbf{q}} G_{\mathbf{q}} e_{\mathbf{q}}$. In this respect, note that in a square numerical mesh of size $L \times L$, quantities such as q_x^2 and q_y^2 must be understood as

$$q_{x,y}^2 \equiv 2 - 2\cos\left(\frac{\pi n_{x,y}}{L}\right) \tag{SM1}$$

with $n_{x,y} = 0, ..., L-1$.

Concerning the form of the local forces $f_i = -dV_i/e_i$, we generate them 'on the flight' in the following way. We start with the value of e_i such that $e_L < e_i < e_R$, where a parabolic potential is defined in terms of e_L, e_R and having an unitary curvature. In concrete, the force on e_i coming from the disordered potential is taken as $f_i = -(e_i - (e_L + e_R)/2)$. As soon as the dynamics makes e_i larger than e_R, we set a new parabolic potential well for e_i by choosing

$$e_L^{new} = e_R \tag{SM2}$$
$$e_R^{new} = e_R + \Delta \tag{SM3}$$

where the Δ is randomly chosen from a flat distribution between 0.5 and 1.5. This is what we call the 'cuspy' potential, since it is composed by a concatenation of parabolic pieces and the transition from one to another produces a discontinuity in the force. In the case of 'smooth potentials' instead, the potential wells are defined and updated in the same form, but the force on each potential well is given by

$$f_i = -\sin(e_i - (e_L + e_R)/2). \tag{SM4}$$

Note that in this form, the value of f_i and also its derivative df_i/e_i are continuous functions of e_i.

In constant stress simulations, the value of σ in Eq.(1) is kept fixed, and the main output of the simulation is the value of $\dot{\gamma}$ [from Eq. (4)]. Such a protocol is used primarily to obtain the flow curves. A second important outcome of these simulations is the distribution of local distances to instability, namely the distribution of the quantities $x_i \equiv e_{Ri} - e_i$. The average minimum value of x_i is used to calculate the θ exponent.

On the other hand, we want to be able to access individual avalanches very close to the critical stress, and collect statistics of size, duration, etc. This is better accomplished by using a quasi-static protocol. To do so, we move away from the fix-stress modelling and modify $G_{\mathbf{q}}$ by defining $G_{\mathbf{q}=0} = -\kappa$, with κ a constant parameter of order one. The equation for the evolution of the average strain is transformed to

$$\dot{\gamma} \equiv \frac{d\overline{e_i}}{dt} = \overline{f_i(e_i)} - \overline{e_i}\kappa + \sigma \tag{SM5}$$

that can be interpreted as a progressive reduction of σ (due to the term $\overline{e_i}\kappa$) as the average position of the interface moves forward. This stress reduction guarantees that any activity in the system will eventually stop, reaching a metastable static configuration. At this point the stress has to be increased again to trigger a new avalanche, and the process can be repeated. The evolution of stress along the simulation is sketched in Fig. SM1. From such a simulation we collect the statistics of avalanche size (S) and stress increments needed to trigger new avalanches $x_{\texttt{min}}$.

It should be mentioned, nevertheless, that quasistatic simulations in the form just described are rather inefficient: The dynamic evolution is continuous and we need to wait until the activity falls below a very low threshold to safely decide that the avalanche has stopped. In the same way, to trigger a new avalanche, the stress has to be increased very slowly to be sure to detect the precise beginning of the new avalanche. In the case of piece-wise parabolic potentials, an accelerated numerical scheme can be implemented as follows. Since we use a poten-

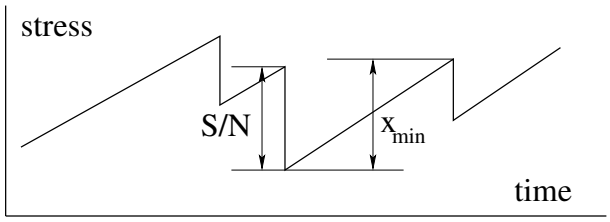

Figure SM1. Sketch of the stress in the system as a function of time in quasistatic simulations. From this kind of plot, statistics of avalanche size (S) and stress increase to destabilize a new avalanche ($x_{\texttt{min}}$) can be collected ($N \equiv L^d$ is the system size).

tial where all parabolic wells have the same curvature, the form of $f_i(e_i)$ is simply given by

$$f_i(e_i) = -(e_i - e_{0i}) \tag{SM6}$$

with $e_{0i} = (e_{Li} + e_{Ri})/2$. Then Eq.(1) can be solved in a single step to obtain the new equilibrium position of the interface. The solution in Fourier space is given by

$$e_{\mathbf{q}} = \frac{e_{0\mathbf{q}}}{1 - G_{\mathbf{q}}}. \tag{SM7}$$

If all the e_i obtained by Fourier-inverting (SM7) lay within their potential well, i.e., $e_{Li} < e_i < e_{Ri}$, then the configuration found is a static solution to the problem. However, if some resulting e_i happen to be outside the range $[e_{Li}, e_{Ri}]$, it means that the corresponding e_{0i} have to be adjusted and Eq.(SM7) solved again to find a new set of e_i. This process is repeated until all e_i are within $[e_{Li}, e_{Ri}]$. At this point the avalanche has stopped. In this scheme one actually loose the true continuous time evolution of the real dynamics, but it results to be computationally much more efficient and–as verified in test cases–it does not show any noticeable differences in the avalanche statistics with respect to the case in which the true dynamics is used.

In all simulations, before collecting statistics of the quantities of interest, we do a sufficiently long equilibration run, and only once the evolution looks stationary (e.g., when we get a stationary value of stress) we start collecting the relevant variables that are reported. The length of the equilibration stage actually depends on the value of ε.