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Criticality and avalanches at the yielding transition of amorphous solids under deformation

Ezequiel Ferrero
Università degli Studi di Milano

Università degli Studi di Napoli Federico II
April 20th 2017

Academic path and chronology

   

Córdoba (ARG) → Bariloche (ARG) → Grenoble (FRA) → Milano (ITA)

- Córdoba National University
  2006-2011 Physics PhD. Advisor: Prof. S.A. Cannas

  Statistical Mechanics of classical spin models

- Bariloche Atomic Center (+stay at LPTMS Orsay)
  2011-2013 postdoc. Drs. A.B. Kolton, S. Bustingorry, A. Rosso

  Disordered elastic systems

- Grenoble Alpes University
  2013-2016 postdoc. Prof. J-L Barrat

  Amorphous solids

- University of Milan
  2017- postdoc. Prof. S. Zapperi

  Metamaterials?

After Milan?….. back to Bariloche (CONICET researcher position)

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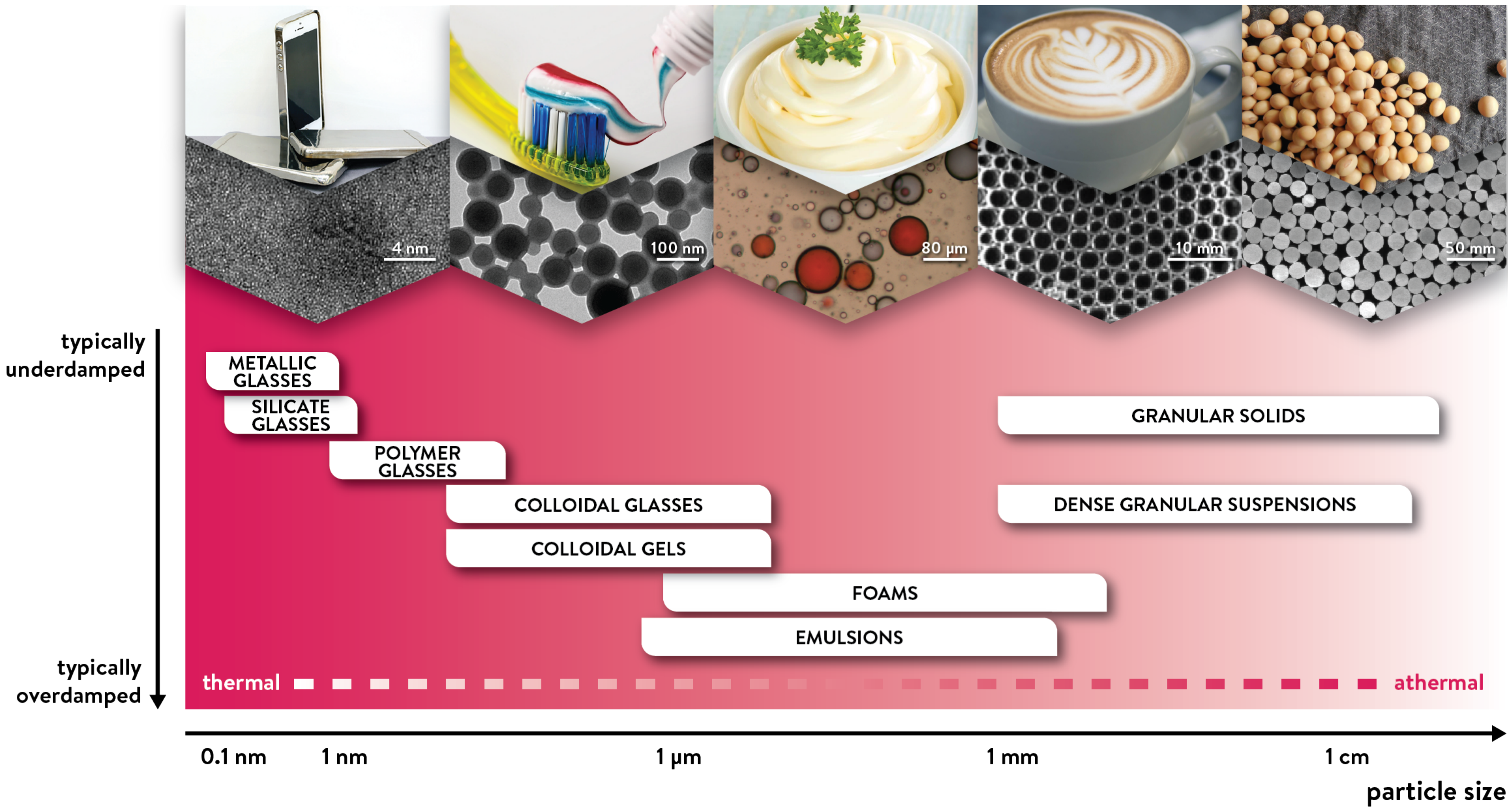
Amorphous materials



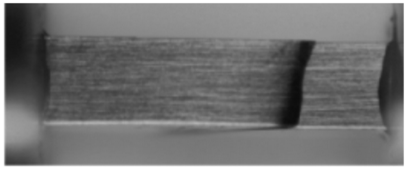
very diverse systems... but they share common features

Structurally ***disordered***

Solid-like (***elastic***) behavior below ***yield stress***

Flow under stress bigger than threshold

Yield stress systems

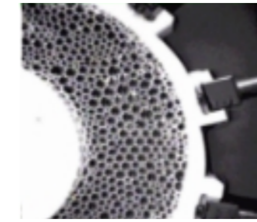


“hard” amorphous

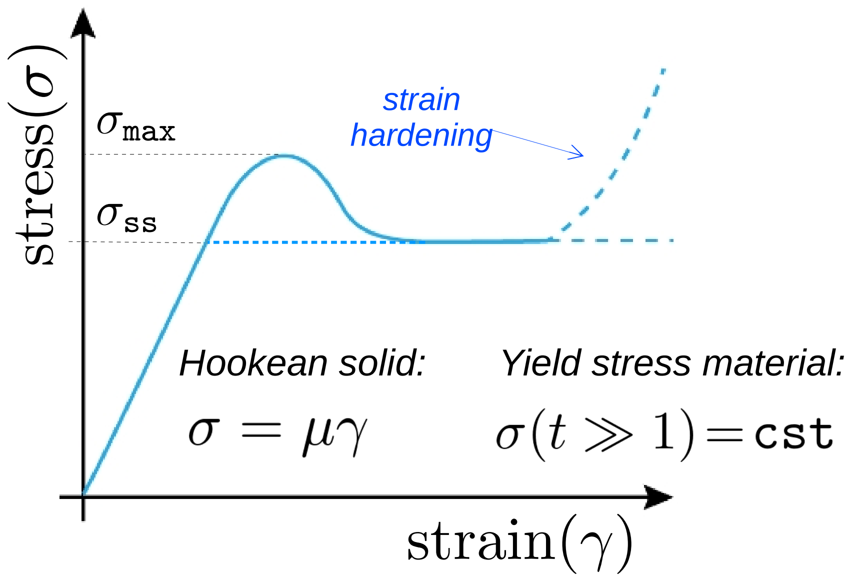
E~100 GPa (BMG)

“soft” amorphous

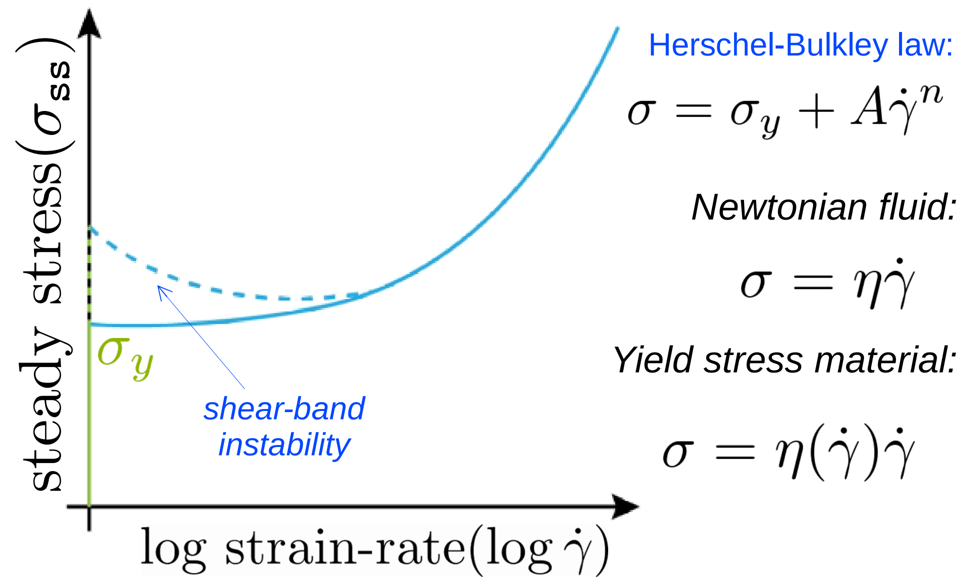
E~100 Pa (Foam)



- ***stress-strain curve***: shear/tensile/compressive (uni/tri-axial) stres vs strain



- ***flow curve***: (shear) stress vs strain-rate in steady shear

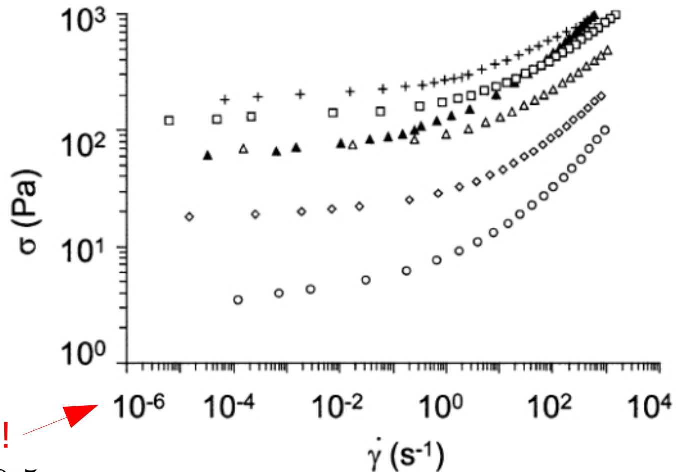


“Yielding transition”: a **dynamical phase transition** between an **elastic solid**-like state and a **plastic flow** state when we overcome a **critical yield stress**.

$$\dot{\gamma} \sim (\sigma - \sigma_y)^\beta \qquad \beta = 1/n$$

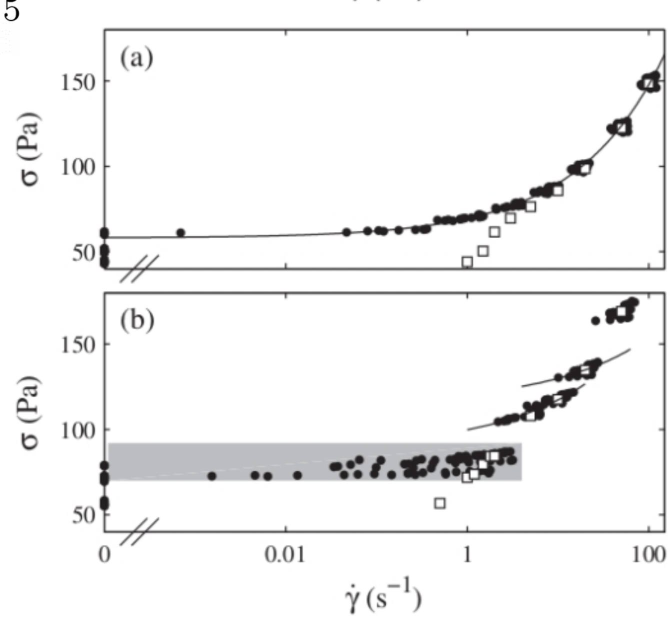
Yield stress systems

Polyelectrolyte gels [1]



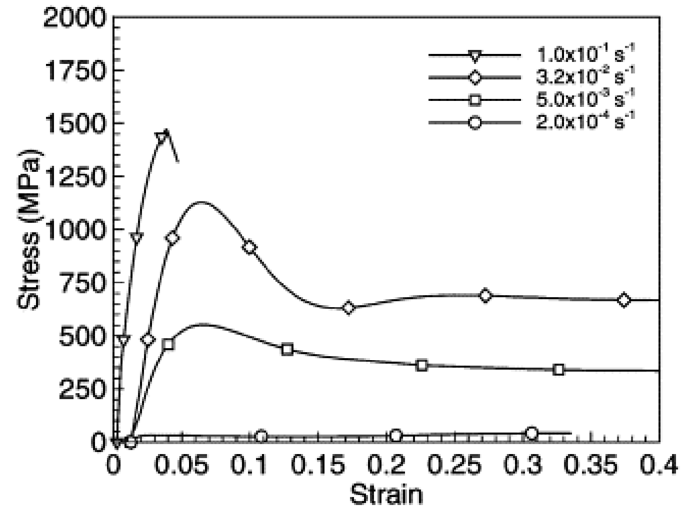
log scale!

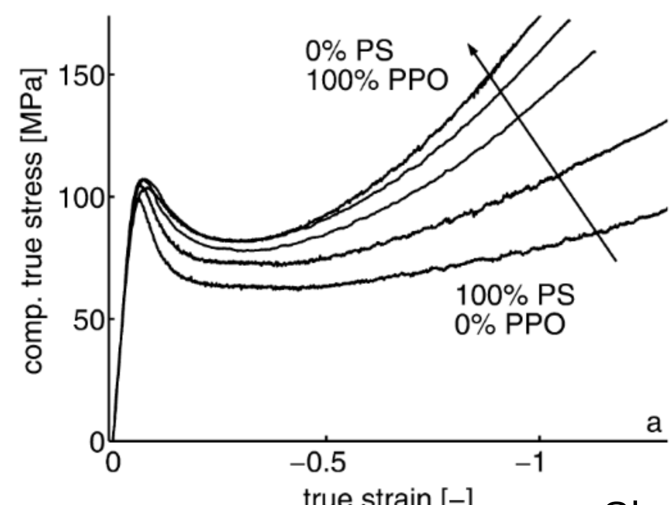
$$\sigma = \sigma_y + A\dot{\gamma}^{0.5}$$



Emulsions [2]

Bulk metallic glasses [3]



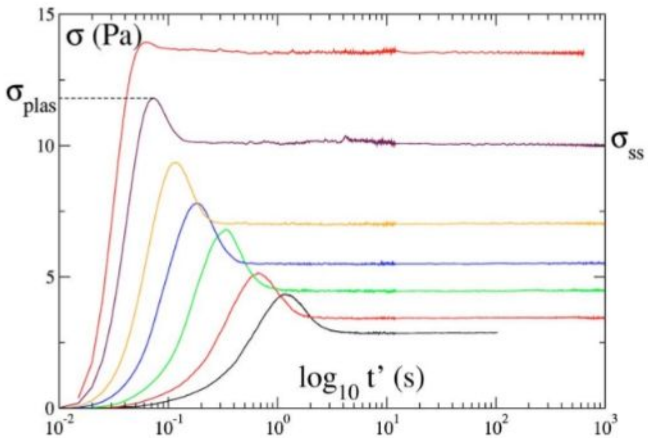


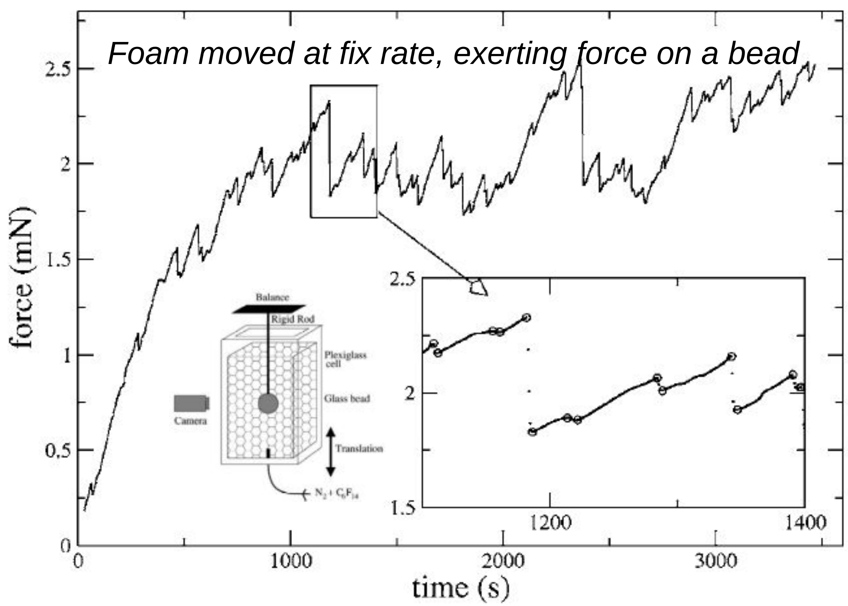
Glassy polymers [4]

[1] M. Cloitre et al. C. R. Physique **4** 221 (2003)
[2] L. Bécu et al. Phys. Rev. Lett. **96** 138302 (2006)
[3] J. Lu et al. Acta Materialia **51** 3429 (2003)
[4] H.G.H. van Melick et al. Polymer **44** 2493 (2003)

Phenomenology: 1. Local rearrangements

Silica colloids sheared at different shear-rates

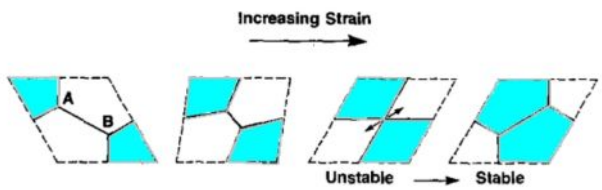


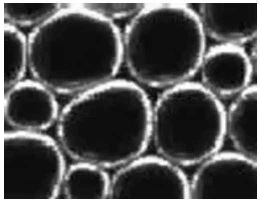


"jerky" aspect of the stress response

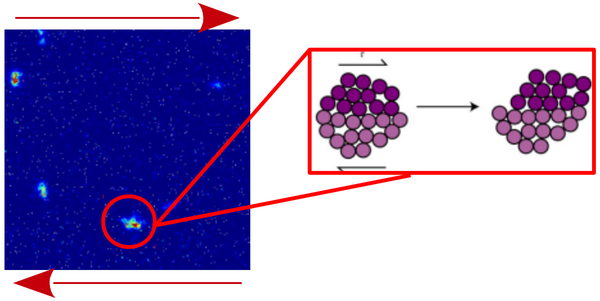
well identified, localized **"plastic events"**

In foams: "T1 event" (4 bubbles)





In general: tens/hundreds of particles involved



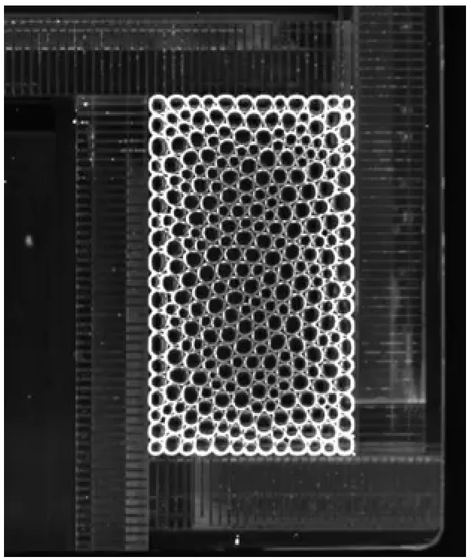
"*plastic event*" = "plastic *rearrangement*" = "*shear transformation*"

C. Derec et al. Phys. Rev. E **67** 61403 (2003)
I. Cantat and O. Pitois *Phys. Fluids* **18** 083302 (2006)

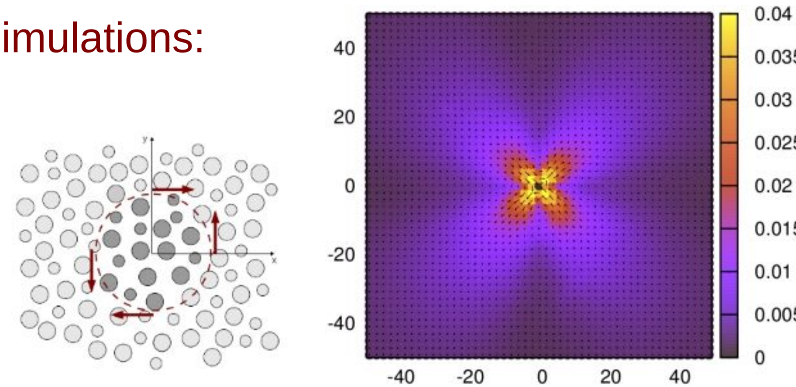
Princen and Kiss, *J Coll. Int. Sci.* **128** 176 (1989)
"T1 event in a densely packed foam" by M. van Hecke, youtube (2014)
A. Nicolas et. al EPJE **37** 50 (2014), A.S. Argon and H.Y. Kuo Mat. Sci. Eng. **39** 101 (1979)

Phenomenology: 2. Medium elastic response

A foam under shear strain



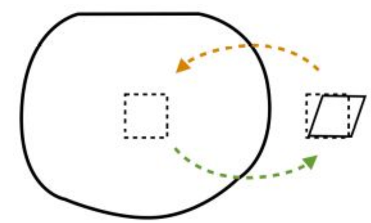
MD simulations:



imposed shear transformation and average displacement field

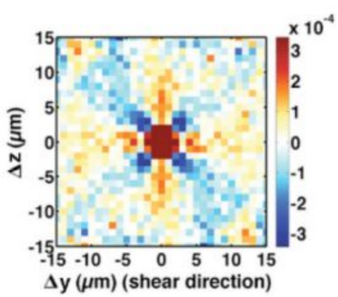
Continuum mechanics:

elastic response to a deformed inclusion

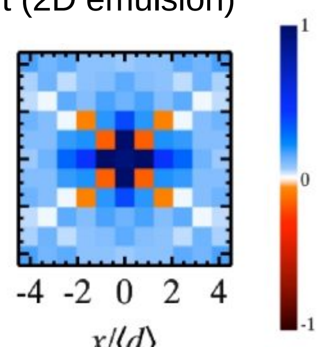


Experimental measurements:

correlations of local strain (sheared colloidal glass)

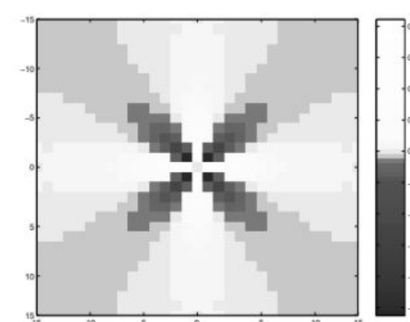


average stress change around an event (2D emulsion)



"Eshelby" propagator for the strain (stress) redistribution

$$G^{2D}(r,\theta) = \frac{1}{\pi r^2} cos(4\theta)$$



Quadrupolar in symmetry,
dipolar in terms of **interaction range**

F. Puosi, J. Rottler, J.-L. Barrat PRE **89** 042302 (2014)
J.D. Eshelby Proc. Roy. Soc. A **241** 376 (1957)
Picard *et al.* EPJE **15** 371 (2004)

"Shearing a 2D foam" by M. van Hecke, youtube (2014)
Jensen et al, PRE **90**, 042305 (2014) Desmond and Weeks, PRL **115**, 098302 (2015)

Coarse-grained Elasto-Plastic Models (EPM)

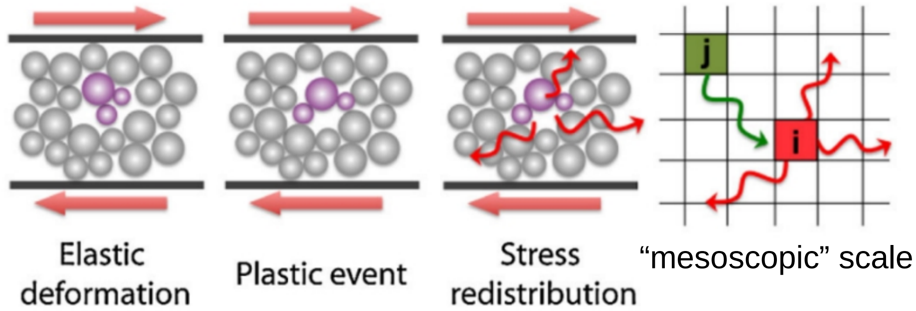


Fig. credit:
Bocquet *et al*. PRL **103**, 036001 (2009)

Common simplifications:

- *Scalar*
- *Athermal*
- *Overdamped*
- *p.b.c.*

Scalar stress field (e.g., shear component) in a grid, representing the stress in **each block**

$$\partial_t \sigma_i(t) = \underbrace{\mu \dot{\gamma}^{\mathtt{ext}}}_{\text{external strain-rate}} - \underbrace{g_0 \partial_t \gamma_i^{\mathtt{pl}}(t)}_{\text{local plastic yield}} + \underbrace{\sum_{j \neq i} G_{(i,j)} \partial_t \gamma_j^{\mathtt{pl}}(t)}_{\text{"mechanical noise" due to plastic activity elsewhere}}$$

external strain-rate — local plastic yield — "**mechanical noise**" due to plastic activity elsewhere

Eshelby **propagator**

$r = |\mathbf{r}_i - \mathbf{r}_j|$

$$G^{2d}_{(i,j)} = \cos(4\theta_{ij})/\pi r^2$$

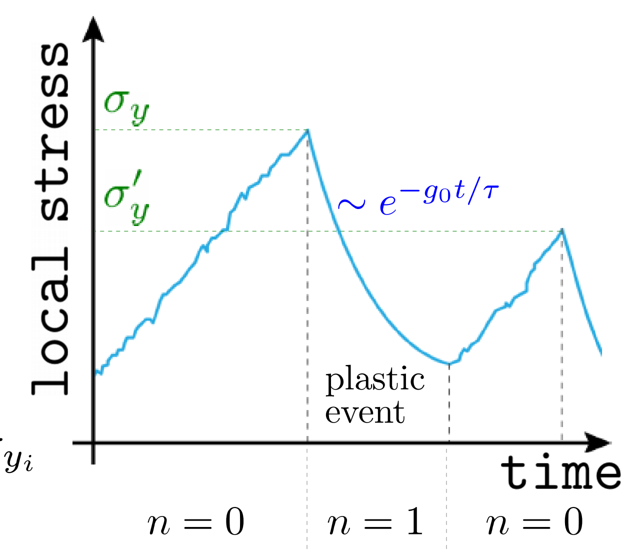
A given plastic strain rate

$$\partial_t \gamma_j^{\mathtt{pl}} = n_j(t) \frac{\sigma_j(t)}{\mu \tau}$$

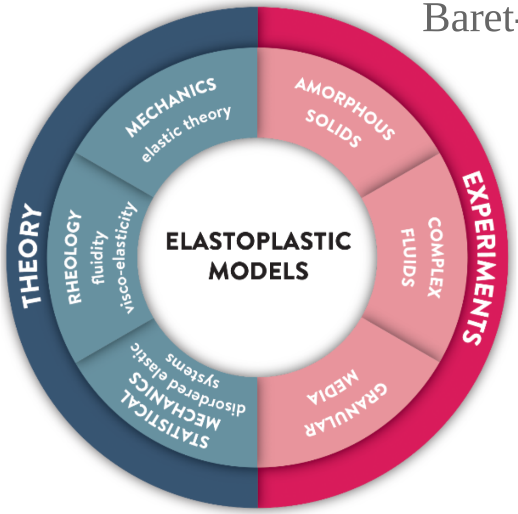
+ Dynamical **rules** for a local "**state variable**"

$n_j(t) = 0$ locally elastic
$n_j(t) = 1$ locally plastic

$$n_i : \begin{cases} 0 \to 1 \text{ typically when } \sigma_i > \sigma_{y_i} \\ 1 \to 0 \text{ e.g., after a time } \tau \end{cases}$$



EPM: phenomenological and toy models

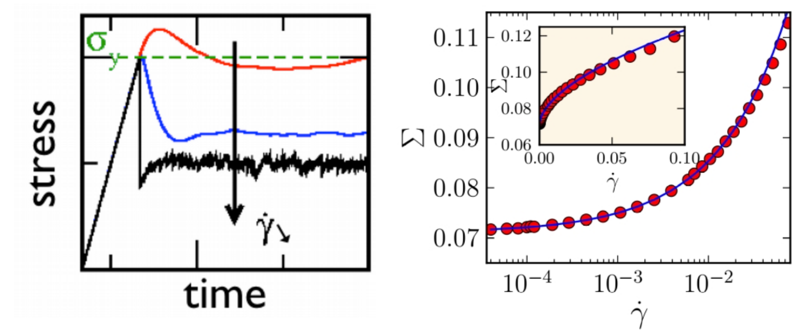


Bulatov-Argon (1994)
Baret-Roux-Vandembroucq (2002)
Picard et al. (2005)
Nicolas et al. (2014)
Ferrero et al. (2014)
Lin et al. (2014)

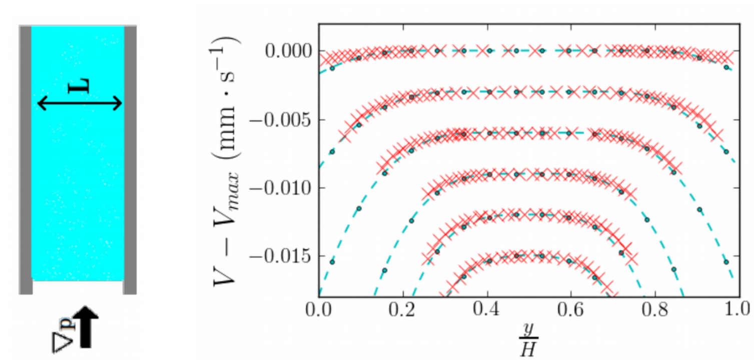
Chen-Bak-Obukhov (1991)
Homer-Schuh (2009)
Onuki (2003)
Jagla (2007)

Also... Talamali (2011), Martens (2012), Budrikis (2015), Papanikolau (2016)

Stress-strain and flowcurves

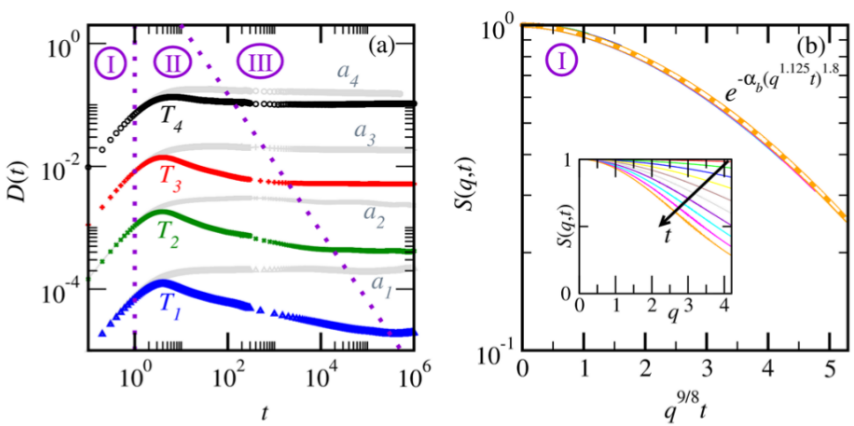


Flow and fluctuations in microchannels



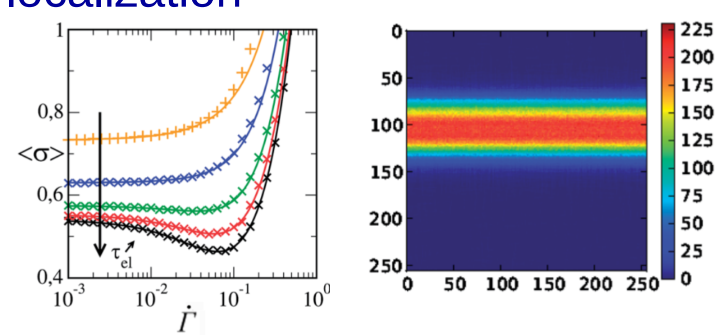
A. Nicolas and J.-L. Barrat, PRL 110, 138304 (2013)

Relaxation in yield-stress systems



EEF, K. Martens, J.-L. Barrat PRL 113, 248301 (2014)

Shear localization



Martens, Bocquet, Barrat, Soft Matter 8, 4197 (2012)

AVALANCHES

Outline:

0) Avalanches in experiments, yielding transition and mean-field approaches.

1) Driving Rate Dependence of Avalanche Statistics and Shapes at the Yielding Transition
Chen Liu, Ezequiel E. Ferrero, Francesco Puosi, Jean-Louis Barrat, Kirsten Martens
Phys. Rev. Lett. **116** 065501 (2016)

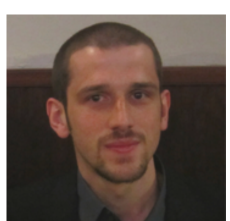
2) Inertia and universality of avalanche statistics: The case of slowly deformed amorphous solids
Kamran Karimi, Ezequiel E. Ferrero, Jean-Louis Barrat
Phys. Rev. E **95**, 013003 (2017)

PSM group

LiPhy Laboratoire interdisciplinaire de Physique

UNIVERSITÉ Grenoble Alpes


Chen


Francesco


Kirsten

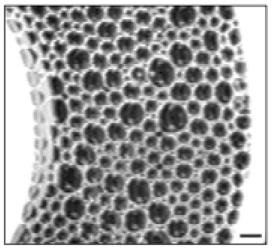

Jean-Louis

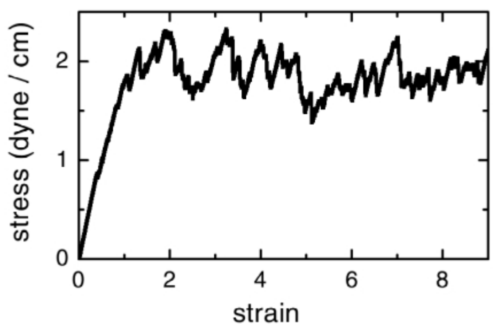

Kamran

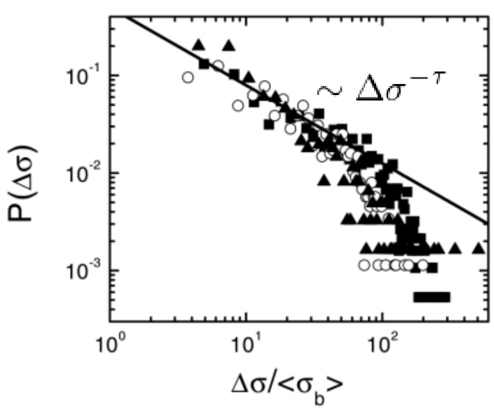
Avalanches: experiments

Plastic flow and stress drops

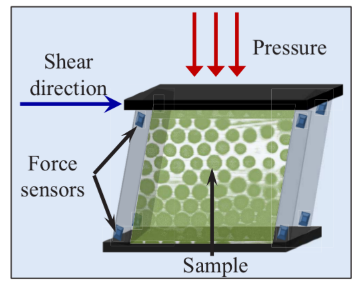
Foams

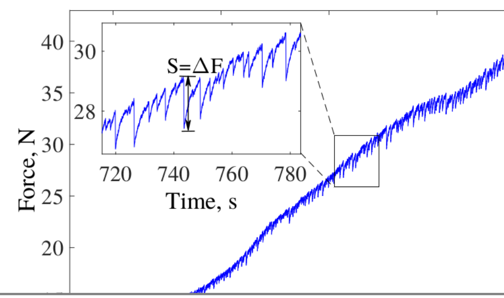


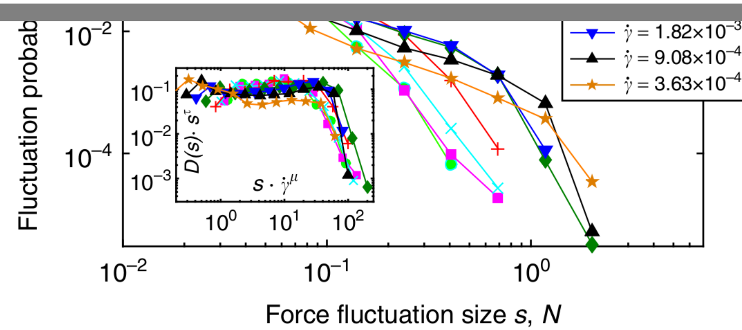




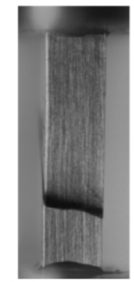
Granular systems

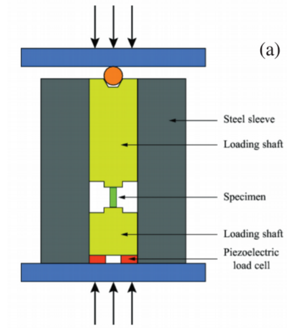


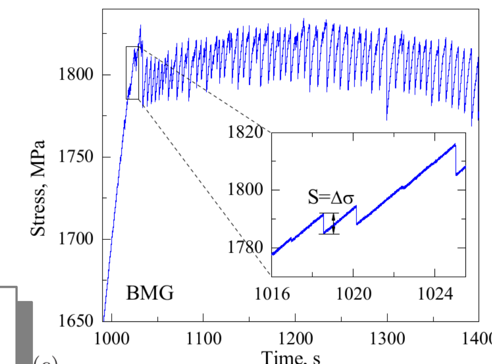


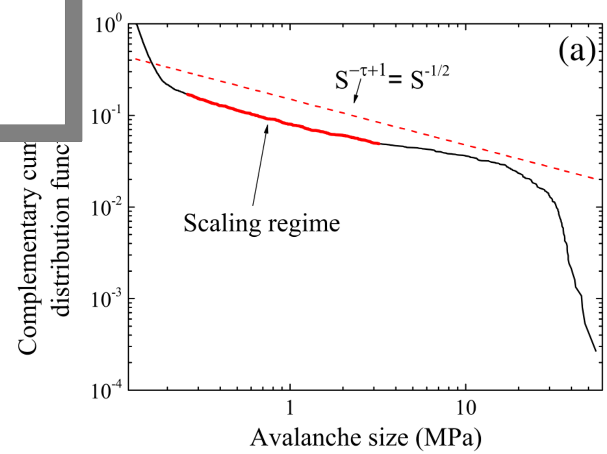


Bulk metallic glasses







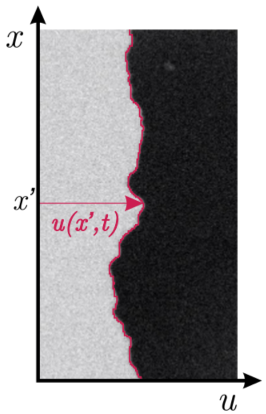


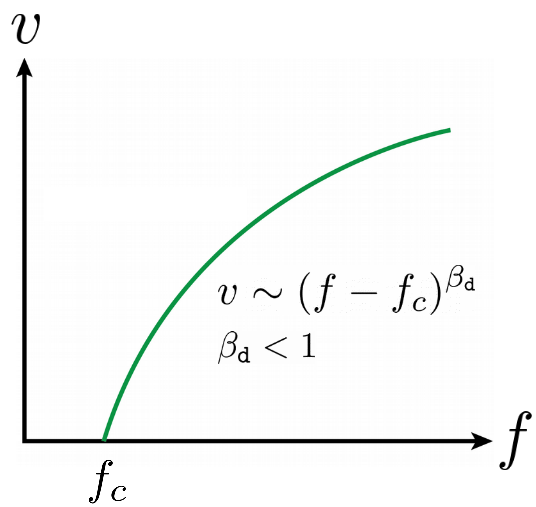
Is this scale freedom related to any kind of “critical” phenomena?
Shall we expect universality?

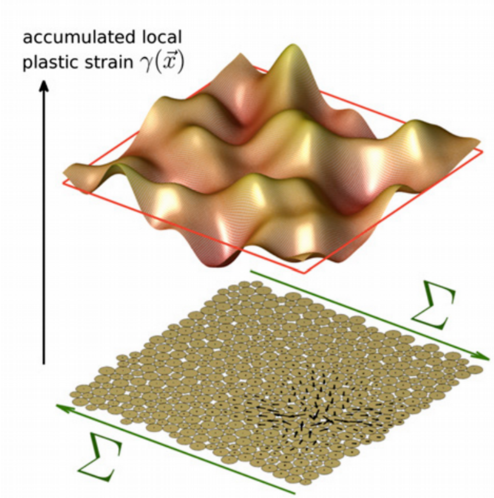
J. Lauridsen et al. PRL **89** 098303 (2002) D. Denisov et al. NatComm **7**:10641 (2016), SciRep **7**:43376 (2017) J. Antonaglia et al. PRL **112** 155501 (2014)

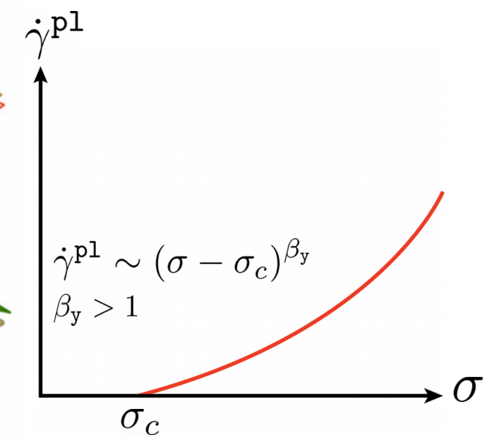
Dynamical phase transitions

depinning and yielding (*similarities*, but also, *important differences*)









Img. Credit: Lin et al., PNAS **111** 14382 (2014)

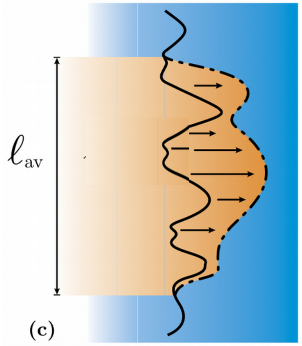
More than **30 years** of research

$$\eta \partial_t u(x,t) = c\partial_x^2 u(x,t) + F_p(u,x) + f$$

Interface is rough and self-affine at threshold $w \sim \ell^{\zeta}$
Divergent length and avalanches:

$$\ell \sim (f - f_c)^{-\nu} \;,\; \nu = \frac{1}{2-\zeta}$$

$$P(S) \sim S^{-\tau} \;,\; \tau = 2 - \frac{2}{d+\zeta}$$



D. Fisher Phys. Reports (1998)
EEF et al., Comp. Rend. Phys. (2013)

Various depinning-analogy proposals* (*long-range elastic interactions case*)

$$\eta \partial_t \gamma_i^{pl} = \mu G_{ij} \gamma_j^{pl} + F_p(\{\gamma_i^{pl}, i\}) + \sigma$$

Divergent length scale and associated avalanche dynamics?

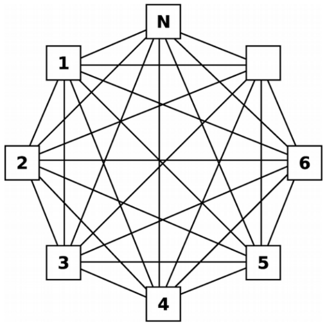
$$\xi \sim |\sigma - \sigma_c|^{-\nu} \;,\; \nu = ? \qquad S = ?$$

Note: collective activity builds not-compact objects

depinning: $d_f \geq d$ yielding: $d_f < d$

*e.g., J. Weiss et al. PNAS **111**, 6231 (2014)
Dahmen et al. PRL **102** 175501 (2009)

Avalanches: mean-field approaches



Fully-connected network of *N* yield stress **blocks** $\sigma_i \quad i = 1, \ldots, N$

1) We **push** blocks towards instability (increase stress) $\sigma_i \to \sigma_i + \delta\sigma$

2) Block ***m*** **reaches the threshold** $(\; \sigma_c = 1 \quad \forall i \;)$

- the stress in *m* drops by a <u>random</u> amount '*u*' $\sigma_m \to \sigma_m - u(1+k)$ $\quad u \sim 1$, $k \to 0$
- all other blocks receive stress "kicks" $\sigma_i \to \sigma_i + \frac{u}{N} + \frac{\eta}{\sqrt{N}}$

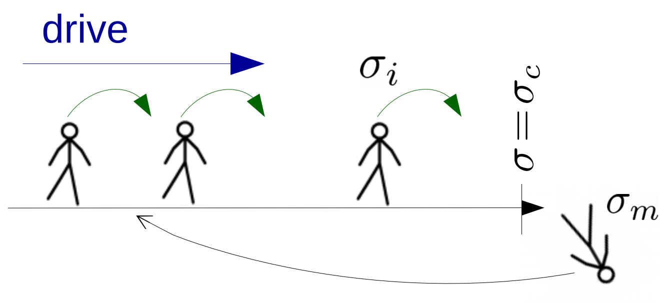
η Gaussian rn, $\langle \eta \rangle = 0$, variance ω

3) We repeat (2) while blocks yield, "avalanche size" is $S = \sum_m u_m$

4) We resume from (1)

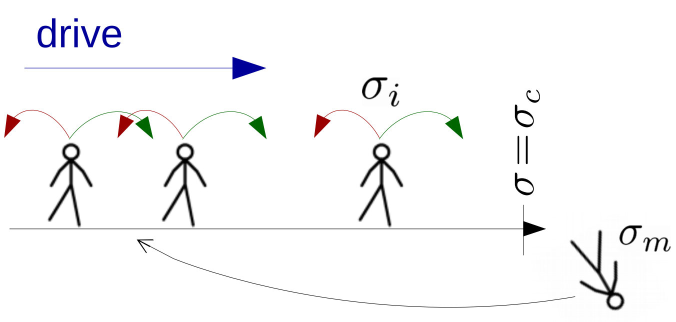
$\omega = 0 \qquad P(\eta) = \delta(\eta)$

All "kicks" are positive (depinning case)



$\omega > 0 \qquad P(\eta) \sim e^{-\eta^2/2\omega}$

"kicks" are positive and negative (yielding case)



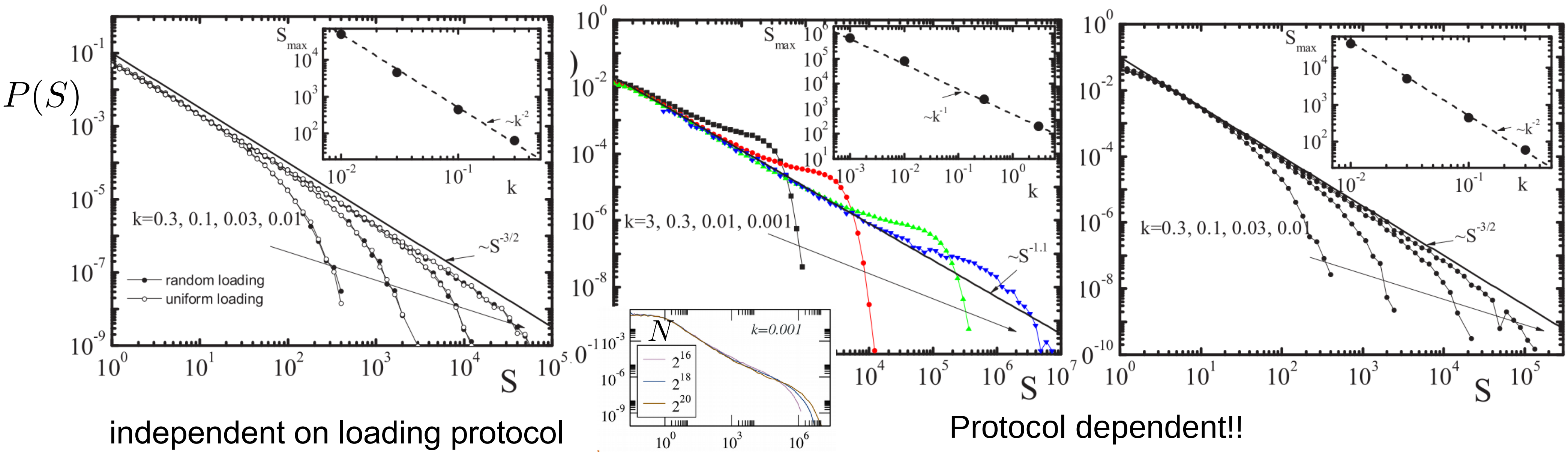
Avalanches: mean-field approaches (by simulation)

E. Jagla PRE **92** 042135 (2015)

$$\sigma_m \to \sigma_m - u(1+k) \qquad P(S) \sim S^{-\tau} f(S/S_{\max}(k)) \qquad \text{critical point: } k \to 0$$

Depinning case $\omega = 0$

Yielding case $\omega > 0$



independent on loading protocol

Protocol dependent!!

Uniform loading

Random loading

$\tau = 1.5$ $\qquad S_{\max} \sim k^{-1/(2-\tau)}$

$\tau \simeq 1.1$ $\qquad S_{\max} \sim (\sqrt{N}/k)^{1/(2-\tau)}$

$\tau = 1.5$ $\qquad S_{\max} \sim k^{-1/(2-\tau)}$

The model which catches the “**non-positive**” nature of the **Eshelby propagator** yields an exponent **different from depinning.** Yet, random triggering restores a constant rate stochastic process for instability and τ=3/2.

Arbitrary overview of mean-field results

Depinning

D.S. Fisher, K.A. Dahmen et al.

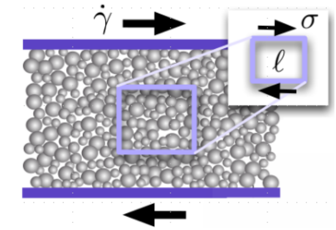
Depinning model for the displacements (plastic "slips") in a solid

$$\eta\partial_t u(\mathbf{r},t) = F + \sigma_{\text{int}}(\mathbf{r},t) - f_R[u,\mathbf{r}]$$

$$\sigma_{\text{int}}(\mathbf{r},t) = \frac{J}{N}\int_{-\infty}^{t} dt'[u(\mathbf{r}',t')] - u(\mathbf{r},t)]$$

$J > 0$ homogeneous positive interaction

Yielding



Hébraud-Lequeux

Evolution equation for the *probability distribution* of local (mesoscopic) stresses

$$\partial_t P(\sigma,t) = -G_0\dot\gamma\partial_\sigma P - \frac{1}{\tau}\theta(|\sigma|-\sigma_c)P + \Gamma\delta(\sigma) + D(t)\partial_\sigma^2 P$$

$D(t) = \alpha\Gamma(t)$ rate of plastic activity: $\Gamma(t) = \frac{1}{\tau}\int_{|\sigma|>\sigma_c} d\sigma P(\sigma,t)$

Unsigned feedback

Yield stress system when $\alpha < \alpha_c$

Depinning			Yielding
$\beta = 1$	*flow curve*	$\dot\gamma \sim (\sigma-\sigma_c)^\beta$	$\beta = 2$ †
$\tau = 3/2$	*avalanches size*	$P(S) \sim S^{-\tau}$	$\tau \simeq 1.1$ ‡
$\theta = 0$	*local distances to threshold distributions*	$P(x) \sim x^\theta$	$\theta = 1$

$x \equiv \sigma^y - \sigma$

† (closer to experiments)

‡ (numerical)

P(x): "density of shear transformations"

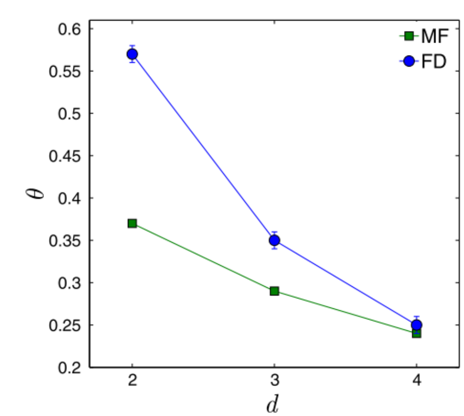
How many incipient STZ are there?

Alternative to HL:

Lin-Wyart (based on **Lemaitre-Caroli**)

Power-law distributed unsigned kicks

$$\theta = \frac{1}{\pi}\arctan\left(\frac{\pi A}{v}\right)$$



D.S. Fisher Ph.Rep. **301**, 113 (1998), K.A. Dahmen et al. PRL **102** 175501 (2009), NPHYS **7** 554 (2011)

P. Hébraud and F. Lequeux PRL **81** 2934 (1998)
E. Agoritsas et al Eur. Phys. J. E **38**, 71(2015)

J. Lin and M. Wyart, PRX **6** 011005 (2016)
A. Lemaitre and C. Caroli arXiv:0609689

Our EP model

C. Liu, EEF, F. Puosi, J-L Barrat, K Martens PRL 116 065501 (2016)

$$\partial_t \sigma(i,t) = \underbrace{\mu\dot{\gamma}^{\texttt{ext}}}_{\text{external strain-rate}} - \underbrace{g_0 n(i,t)\frac{\sigma(i,t)}{\tau}}_{\text{local plastic yield}} + \underbrace{\sum_{j\neq i} G(i,j)n(j,t)\frac{\sigma(j,t)}{\tau}}_{\text{“mechanical noise” due to plastic activity}}$$

external strain-rate — local plastic yield — “**mechanical noise**” due to plastic activity

Eshelby **propagator**

$$G_{2d}(i,j) = \cos(4\theta_{ij})/\pi r^2 \qquad r = |\mathbf{r}_i - \mathbf{r}_j|$$

Simplifications:

- *Scalar*
- *Athermal*
- *Overdamped*
- *p.b.c.*

Euler integration + pseudospectral method (intensive use of FFT)

$$\hat{G}_{2d} = -4\frac{q_x^2 q_y^2}{q^4}$$

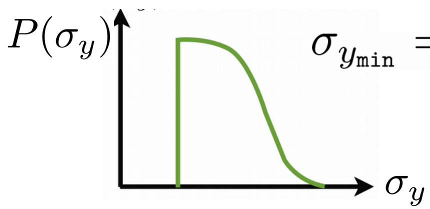
$$\hat{G}_{3d} = -4\frac{q_x^2 q_y^2 + q_z^2 q^2}{q^4}$$

Massively parallel implementation on GPUs

rules for local **state variable**

$$n_i : \begin{cases} 0 \to 1 \text{ when } \sigma_i > \sigma_{y_i} \\ 1 \to 0 \text{ when } \int dt' |\dot{\gamma_i}^{\text{tot}}(t')| \geq \gamma_c \end{cases}$$

new σ chosen after yielding

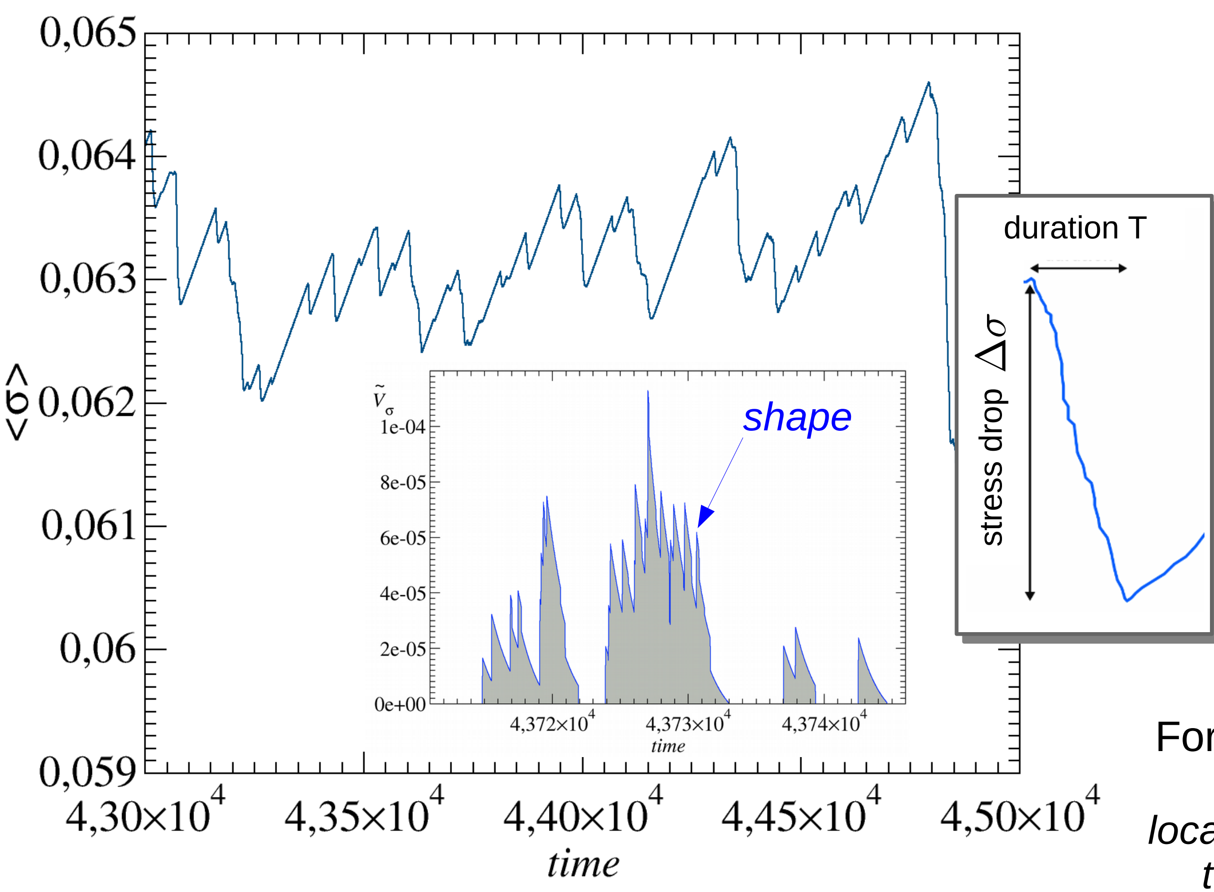


$$\sigma_{y_{\texttt{min}}} = \mu\gamma_c$$

A. Nicolas, K. Martens, J-L Barrat EPL **107** 44003 (2014)

Avalanches: Methods

For different imposed strain rates...



Observables:

For **each event** (stress-drop *Δσ*) we compute:

duration $T = t_{\texttt{end}} - t_{\texttt{start}}$

size $S \equiv L^d \Delta\sigma$

shape $\tilde{V}_\sigma(t) = \left[-\frac{d\langle\sigma\rangle(t)}{dt} \right]_{t_{\texttt{end}}}^{t_{\texttt{start}}}$

For several **configurations** in time

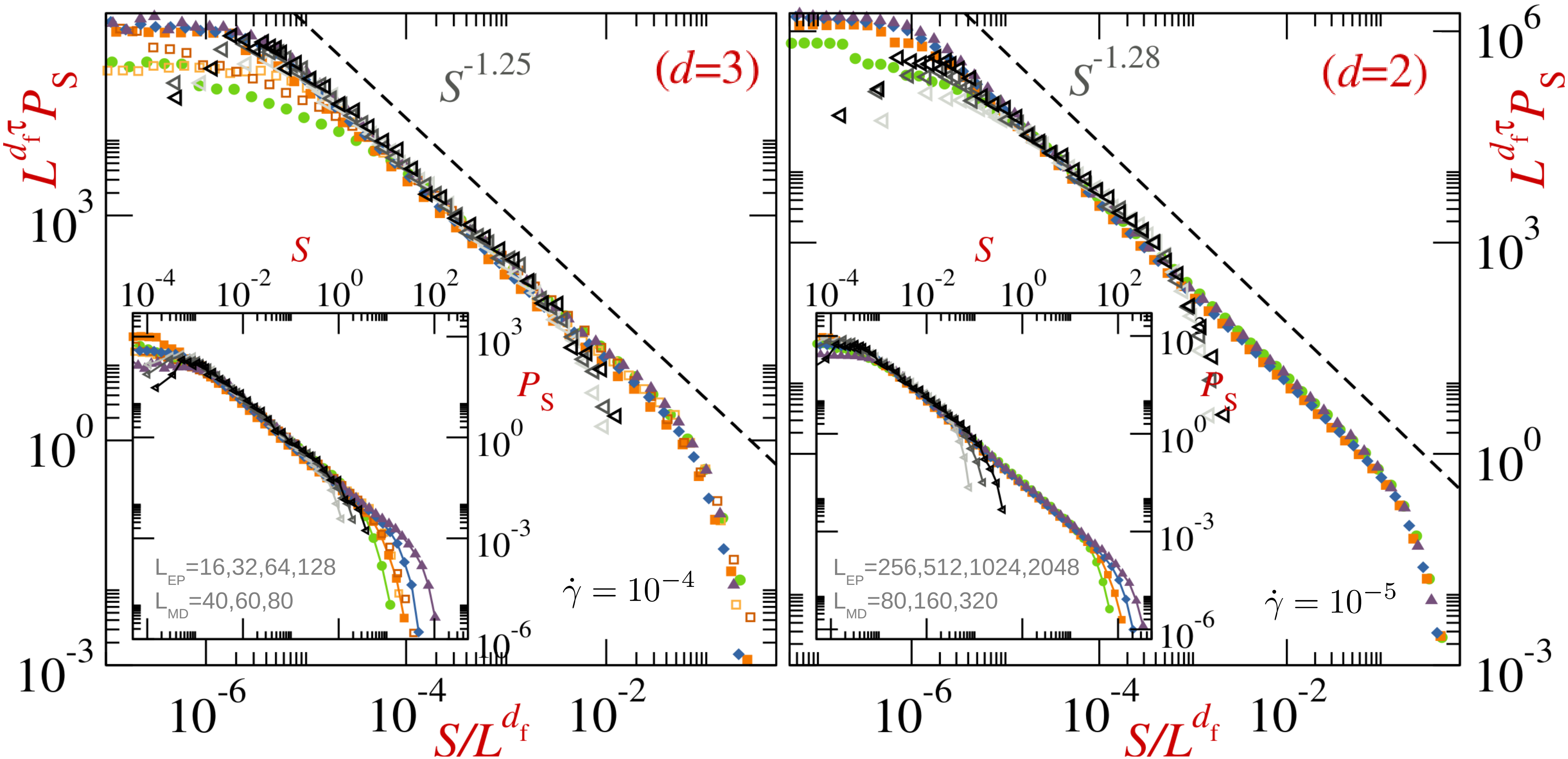
local distances to threshold $\{x\}\ ,\ x_i \equiv \sigma_i^y - \sigma_i$

C. Liu, EEF, F. Puosi, J-L Barrat, K Martens PRL 116 065501 (2016)

Results

Stress drop size distribution at very low shear rates

...for different system sizes, comparing with <u>quasistatic</u> **MD** simulations (grayscale triangles)



$\tau_{3D} \simeq 1.25 \neq \tau_{\mathtt{MF}}^{\mathrm{dep}}$

$d_f^{3D} \simeq 1.3$

$$P_S \propto S^{-\tau} f(S/S_c) \ , \ S_c \simeq L^{d_f}$$

$\tau_{2D} \simeq 1.28$

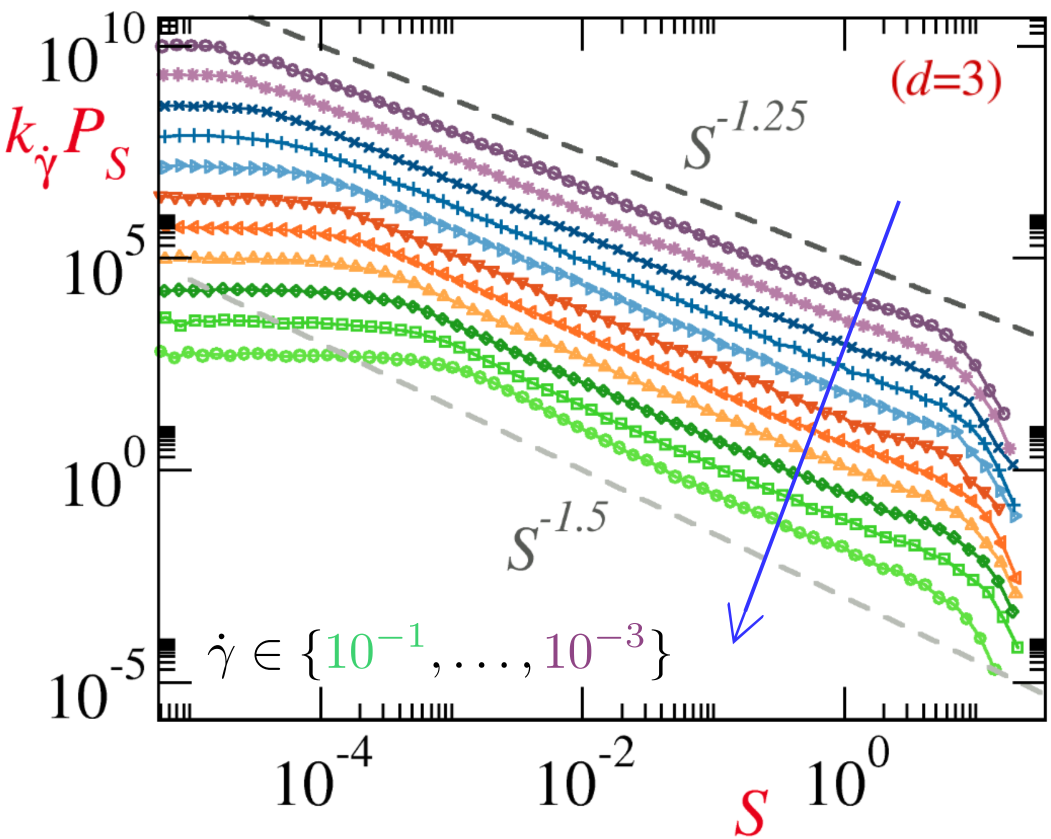
$d_f^{2D} \simeq 0.9$

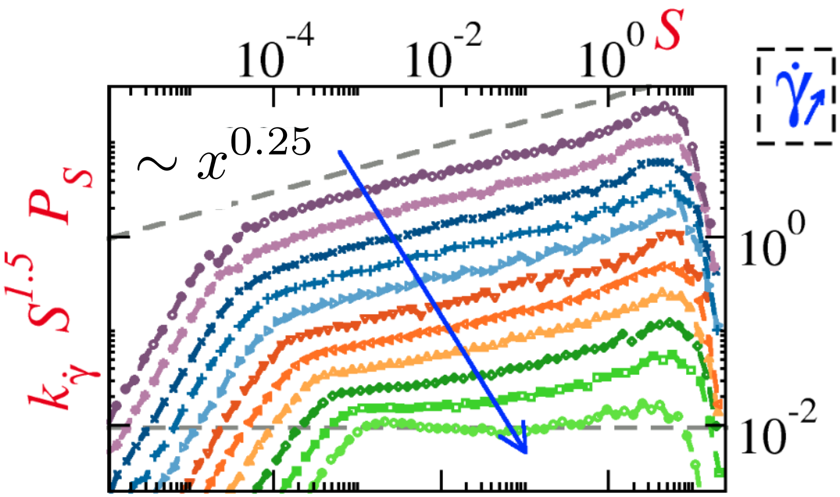
scaling factor d_f: "fractal dimension". *Slip-line* avalanche geometry

Talamali et al. PRE **84** 016115 (2011) $\tau_{2D} = 1.25 \pm 0.05$ (EPM quasistatic) Salerno and Robbins PRE **88** 060206 (2013)

Results

Size distributions and crossover to mean-field behavior





- Large strain-rates **"randomizes"** the stress signal, **by overlapping** uncorrelated plastic activity.
- **Crossover** to **"random triggering" (or depinning)** mean-field exponent when we **go away** from the yielding point

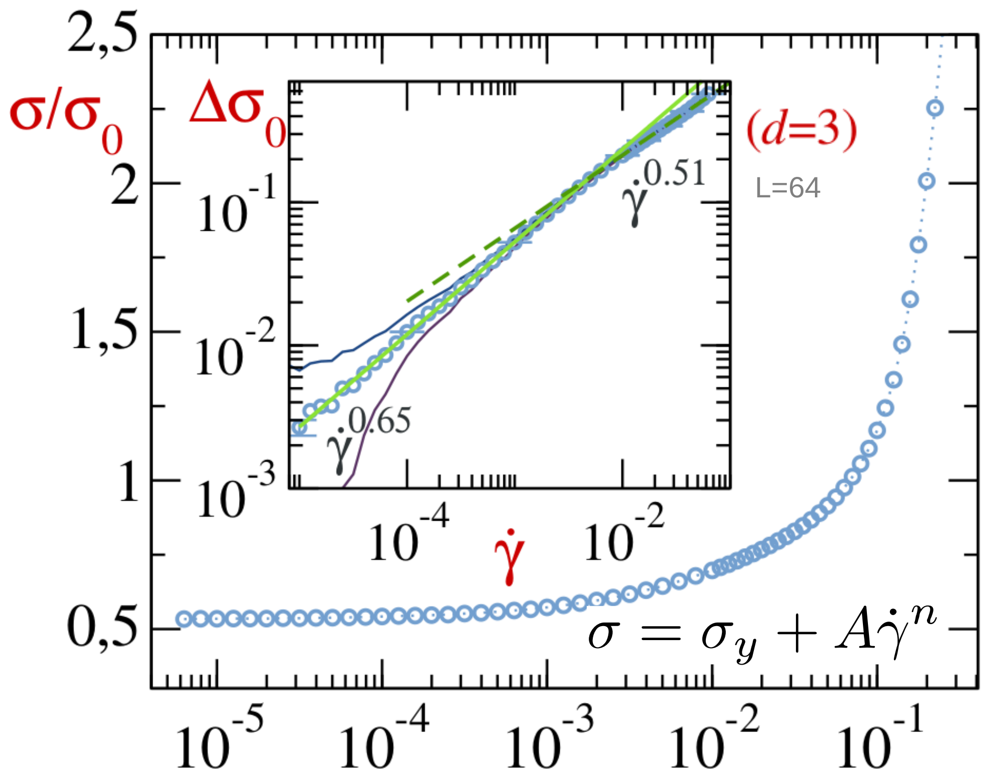
$$\tau : 1.25 \to 1.5$$

(curves arbitrarily shifted by $k_{\dot\gamma}$)

Be ξ^d the size of a "correlated event", with $\xi \sim |\langle\sigma\rangle - \sigma|^{-\nu} \sim \dot\gamma^{-\nu/\beta}, \;\; \beta = 1/n$
In this regime, many events may "fit" in L^d. $\Delta\sigma$ results from this superposition.
$S \equiv \Delta\sigma L^d$ cutoff is controlled by L

Results

Flow-curve and crossover to mean-field "randomized" behavior



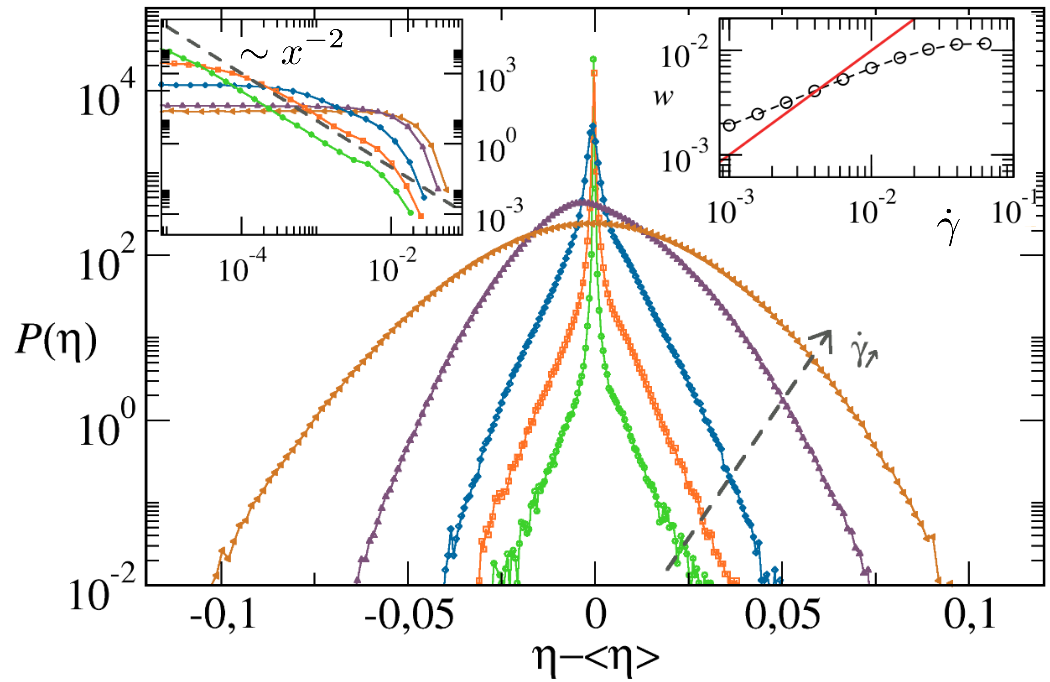
$\Delta\sigma_0 = \frac{\sigma - \sigma_y}{\sigma_y}$ $\dot{\gamma}$

The "yielding transition" $\dot{\gamma} \sim (\sigma - \sigma_y)^{\beta}$

$\beta = 1/n > 1$

- β **crosses over toward the Hébraud-Lequeux mean-field** prediction when $\dot{\gamma}$ increases.

$$\beta \simeq 1.54 \to 2$$



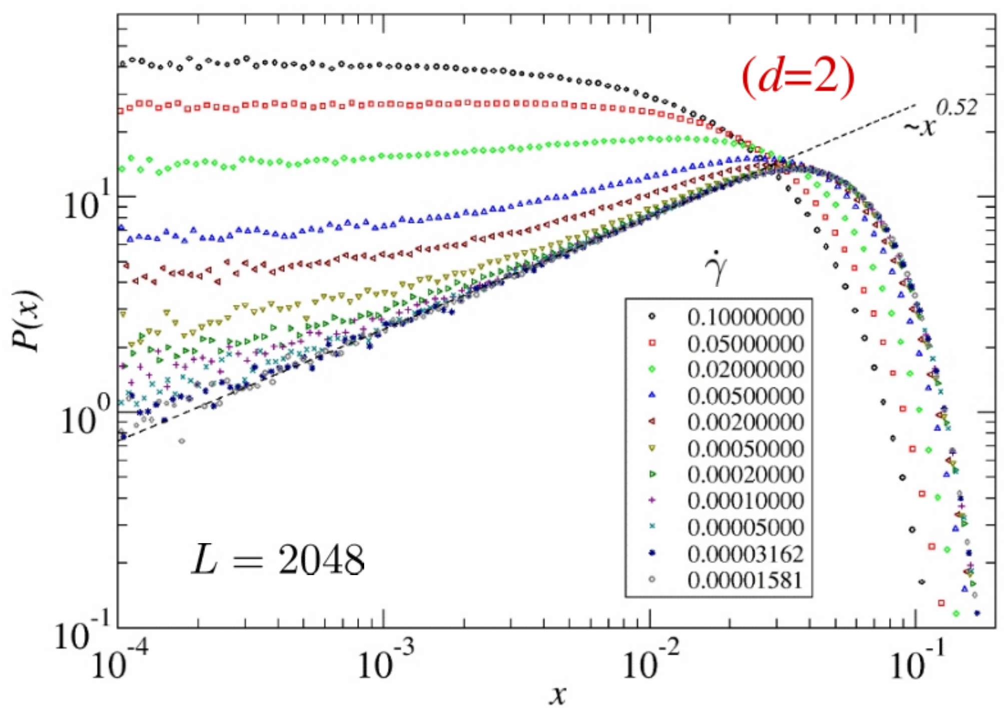
"Mechanical noise" ⟶ η_i

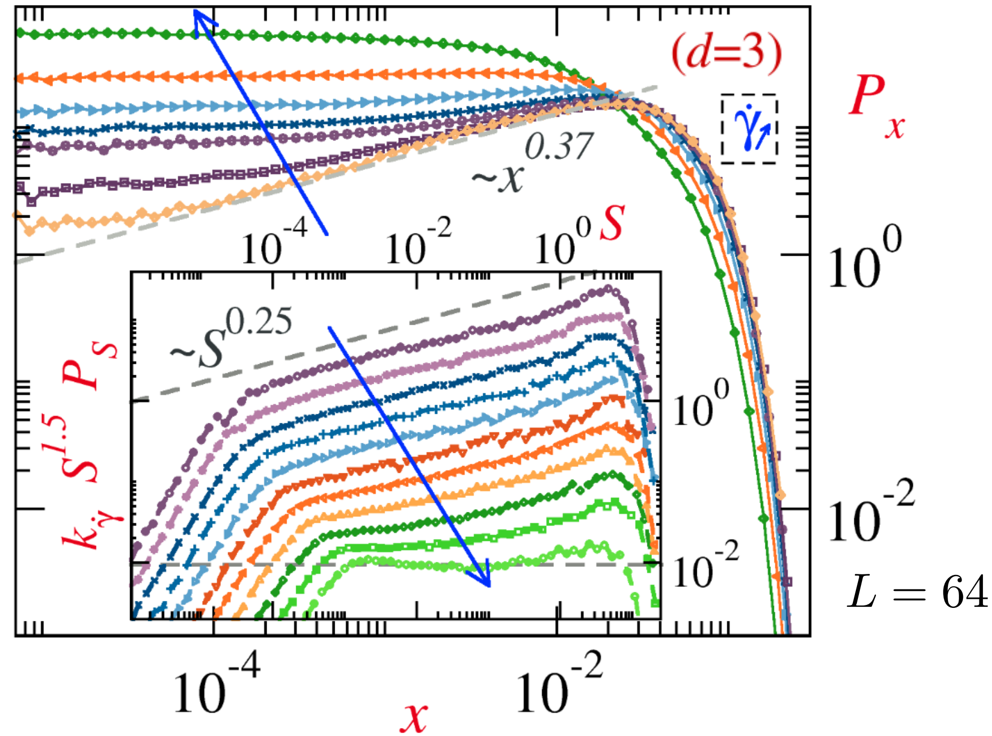
$$\partial_t \sigma_i = \mu\dot{\gamma} - g_0 n_i \frac{\sigma_i}{\tau} + \underbrace{\textstyle\sum_{j\neq i} G_{ij} n_j \frac{\sigma_j}{\tau}}$$

- At large rates, noise distribution turns Gaussian → **loss of non-trivial correlations**
- Variance grow slower than linear with $\dot{\gamma}$ → **drift dominates when $\dot{\gamma} \gg 1$**

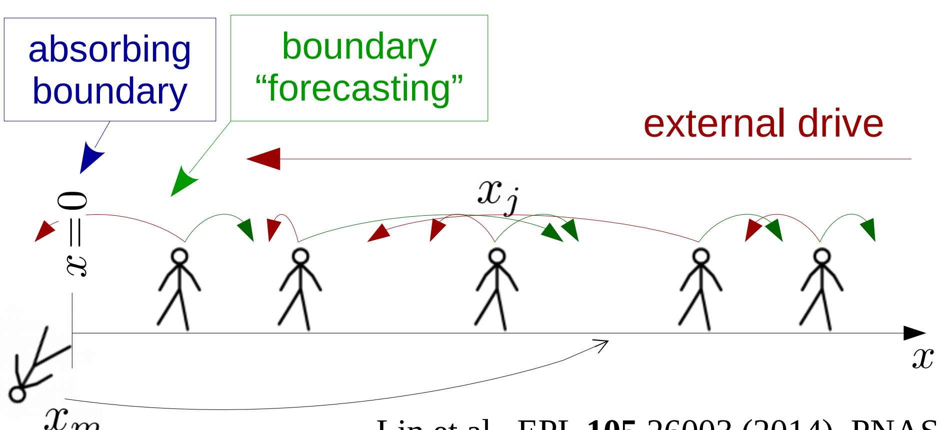
Results

Distribution of local distances to threshold (or "density of shear transformations")





Local **distances to threshold** $x \equiv \sigma_y - \sigma$



We expect: *"marginal stability"* pseudo-gap *(M. Wyart and co.)*

$$P_x \sim x^{\theta} \qquad \theta > 0 \qquad \theta_{2D}^{qs} \simeq 0.57 \quad \theta_{3D}^{qs} \simeq 0.35$$

We observe:

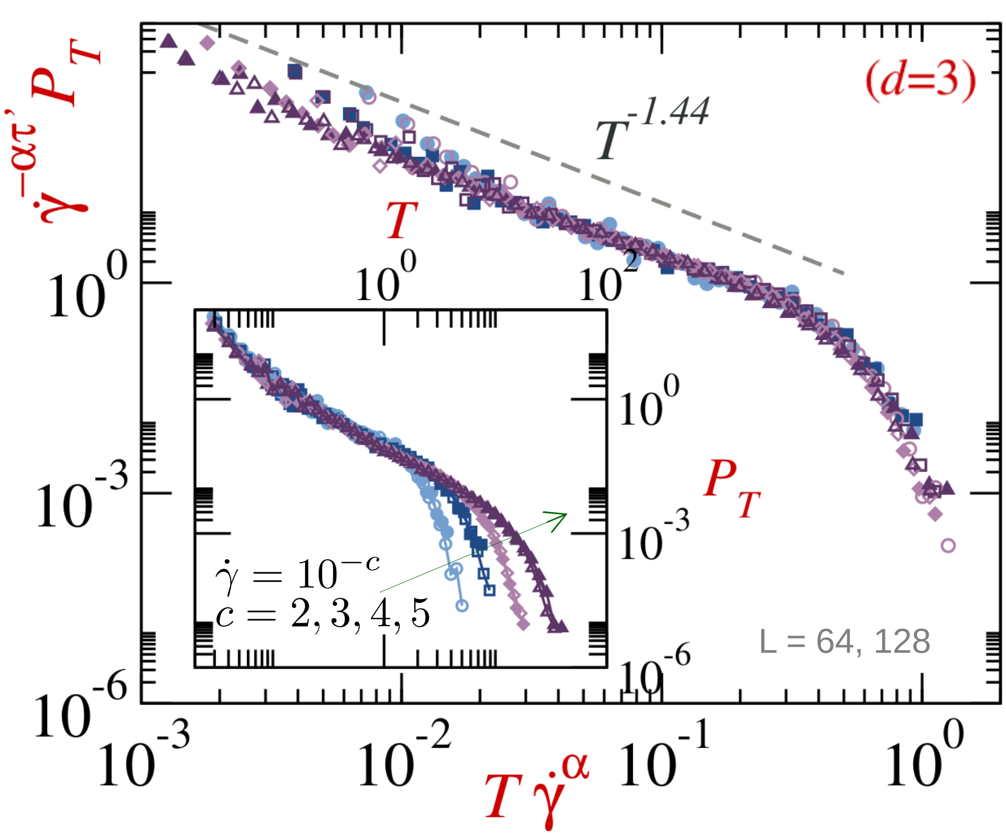
At $\dot{\gamma} \to 0$ $\qquad \theta_{2D} \simeq 0.52 \quad \theta_{3D} \simeq 0.37$

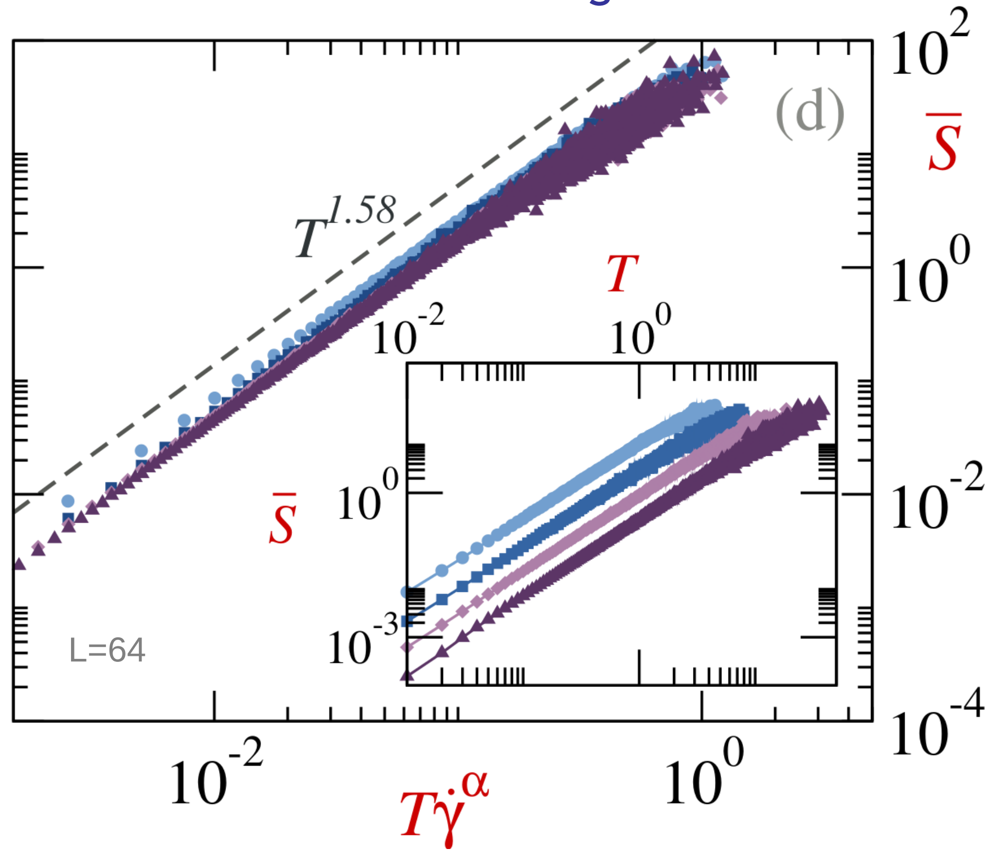
When $\dot{\gamma} \gg 0$ $\qquad \theta \to \theta^{dep} = 0$

Lin et al., EPL **105** 26003 (2014), PNAS **111** 14382 (2014), Müller & Wyart, Annu. Rev. Condens. Matter Phys. **6** 9 (2015)

Results

Stress drop duration distribution and size-duration scaling





$$P_T \propto S^{-\tau'} g(T/T_c)\ ,\ T_c \simeq \dot\gamma^{-\alpha}$$

$$\tau'_{3D} \simeq 1.44 \quad \alpha_{3D} \simeq 0.30$$
$$\tau'_{2D} \simeq 1.41 \quad \alpha_{2D} \simeq 0.38$$

Who is α?

$$T \sim \xi^z \sim \dot\gamma^{-\nu z/\beta} \quad \Rightarrow \alpha \equiv \nu z/\beta = 1 - 1/\beta$$
$$\xi \sim |\langle\sigma\rangle - \sigma_c|^{-\nu} \qquad T_c \sim \dot\gamma^{n-1}$$

We expect: $S \sim T^\delta \qquad \delta = d_f/z$

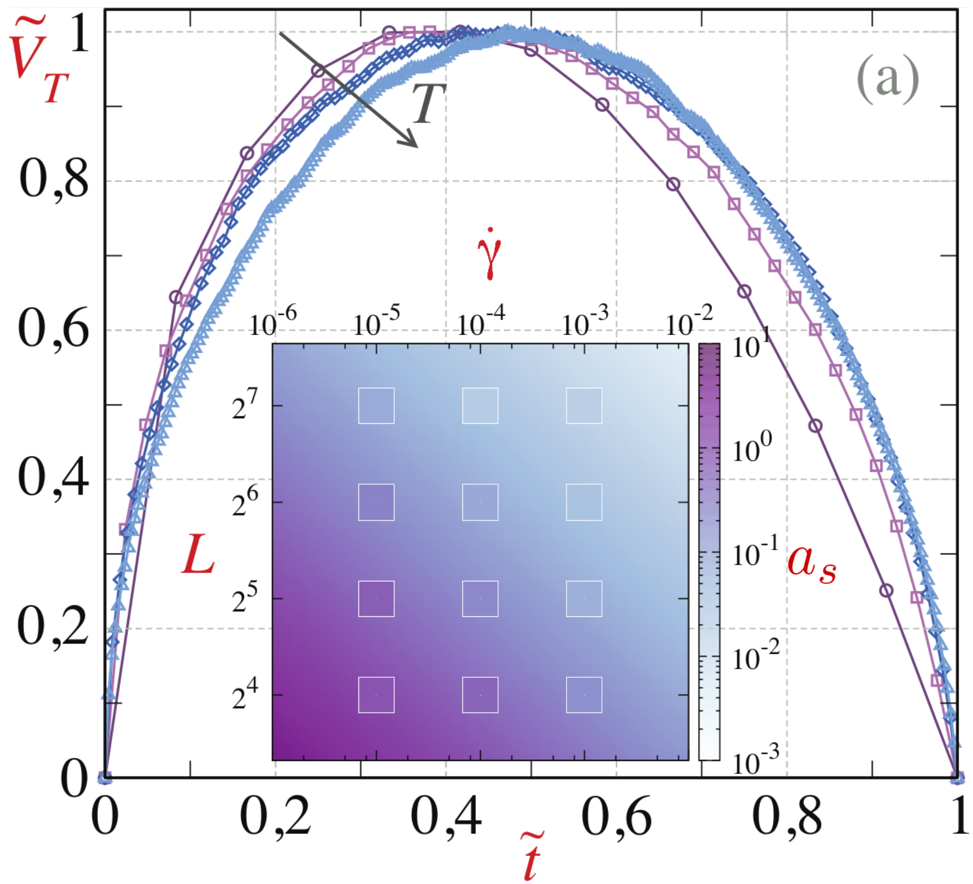
We observe: $S \simeq C(L, \dot\gamma) T^\delta$

$$\frac{S}{L^{d_f}} \sim \left(\frac{T}{\dot\gamma^{-\alpha}}\right)^\delta \qquad \delta \simeq 1.58$$

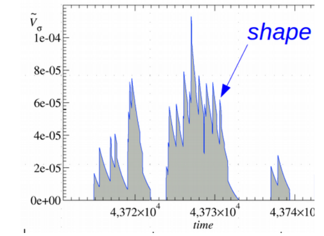
Again, exponents **differ** from **MF depinning**

Results

Stress drop shapes (averaged at fix T)



Recall: $\tilde{V}_\sigma(t) = \left[-\frac{d\langle\sigma\rangle(t)}{dt}\right]_{t_{\mathrm{end}}}^{t_{\mathrm{start}}}$



Normalized shape for a drop of duration T:

$$\tilde{V}_T(t) = V_T(t)/\max_t(V_T(t)) \qquad \tilde{t} = t/T$$

Fitting function*:

$$\tilde{V}_T(\tilde{t}) \propto B(\tilde{t}(1-\tilde{t}))^c(1-a_s(\tilde{t}-0.5))$$

$B \sim T^c$ $\quad c = \delta - 1$ $\quad$ holds $\quad B \sim T^{0.6}$

Inset: "**asymmetry**" parameter

- Drops of **short durations** show a clearly **asymmetric** shape
- For **large *T*** stress drops shapes become more **symmetric**.
- **Superposition** of "individual" avalanches due to **finite strain-rate**.

*L. Laurson et al. Nature Comm. **4** (2013)

Summary 1/2

C. Liu, EEF, F. Puosi, J.-L. Barrat, K. Martens
Phys. Rev. Lett. **116** 065501 (2016)

- Our results reinforce the idea of a **non-MF-depinning universality class** for the yielding transition below d=4.

- **Departing from** the **yielding** point, at **finite shear rates**, the rise of many independent regions with yielding activity randomizes the response and draw exponents **closer to MF expectations**.

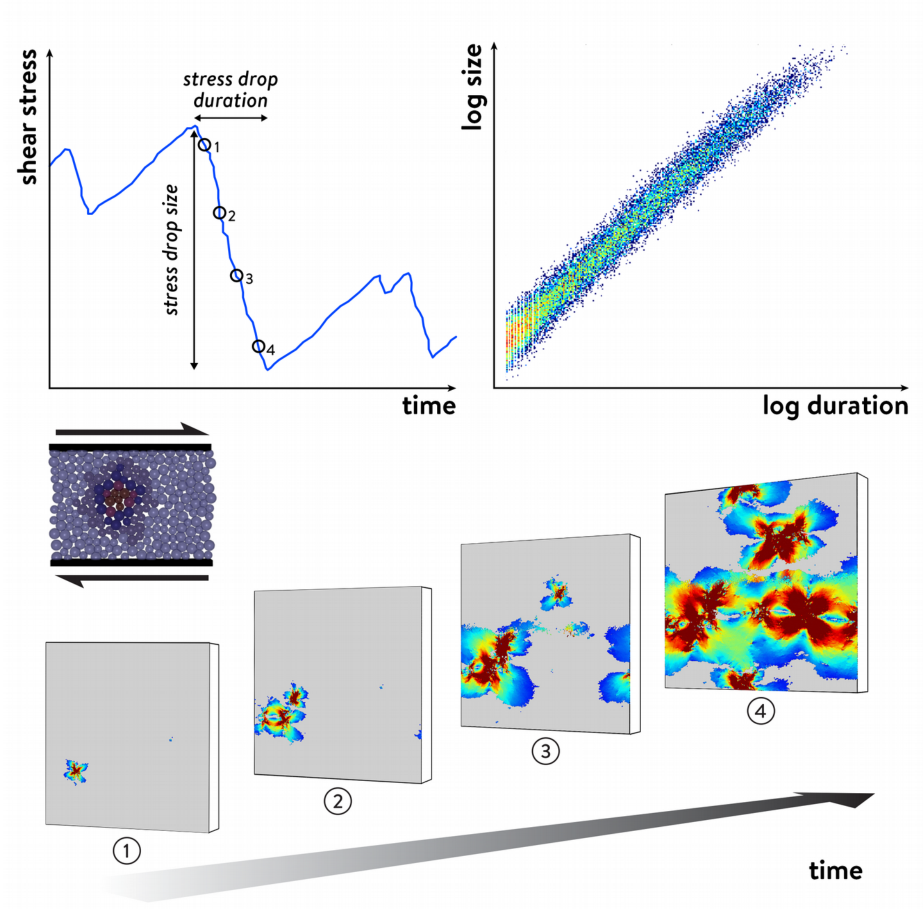
- ***The density of STZs crosses over*** from yielding marginal stability $P(x)\sim x^{\theta}$ to depinning-like $P(x)\sim cst.$ when increasing the external strain rate.

- **Scaling relations** hold within exponent's error bars

$$\beta = \nu(d - d_{\mathrm{f}} + z)$$

$$\nu = 1/(d - d_{\mathrm{f}})$$

$$\tau = 2 - \frac{\theta}{\theta+1}\frac{d}{d_f}$$



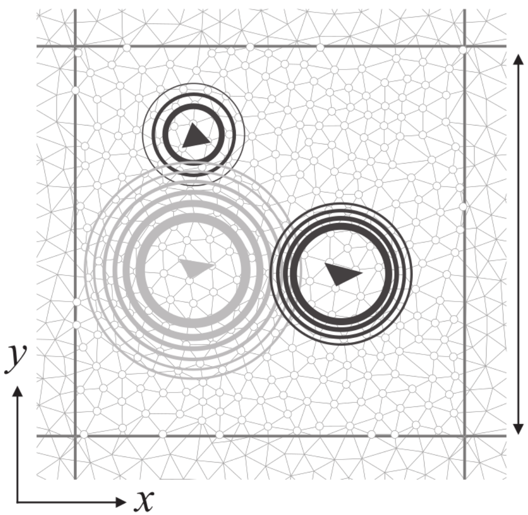
J. Lin, E. Lerner, A. Rosso, M. Wyart, PNAS **111** 14382 (2014)

Finite Elements Method approach

To account for inertial effects

K. Karimi, EEF, J-L Barrat, PRE 95, 013003 (2017)

Irregular 2d lattice, **tensorial** model



Local yielding
+
Elastic waves

Continuum mechanics e.o.m.:

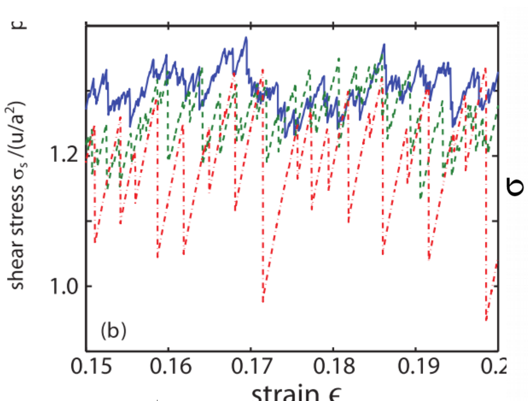
$$\rho \ddot{\mathbf{u}}(\mathbf{r},t) = \boldsymbol{\nabla}.\boldsymbol{\sigma}(\mathbf{r},t)$$

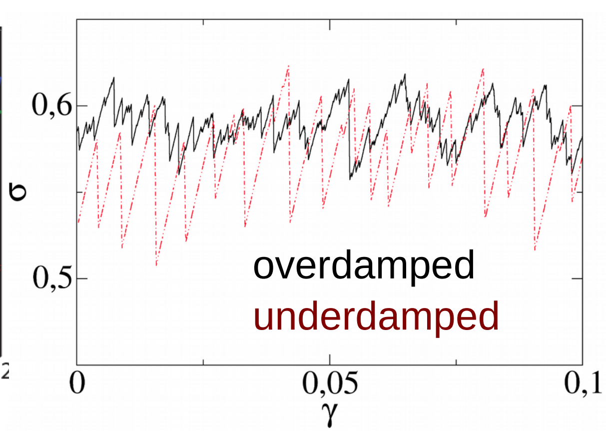
$\mathbf{u}(\mathbf{r},t)$: displacement field

$\boldsymbol{\sigma}$: internal stress

$$\boldsymbol{\sigma} = \underbrace{\mu(t)\nabla\mathbf{u}}_{\text{restoring}} + \underbrace{\eta\boldsymbol{\nabla}\dot{\mathbf{u}}}_{\text{dissipative}}$$

$$\Gamma = \frac{\tau_d^{-1}}{\tau_v^{-1}} = \frac{\eta/(\rho a^2)}{\sqrt{\mu/\rho a^2}}$$: dissipation coefficient





Lower η, more inertial

+ EP rules: $n : 0 \underset{\text{after elapsed } \tau_{\text{on}}}{\overset{\text{when } \sigma > \sigma_y}{\rightleftarrows}} 1$

$$\mu(t) = \begin{cases} 0 & \text{while } n = 1 \\ \mu & \text{otherwise} \end{cases}$$

Molecular dynamics

K.M. Salerno & M. Robbins PRE 88, 062206 (2013)

Results

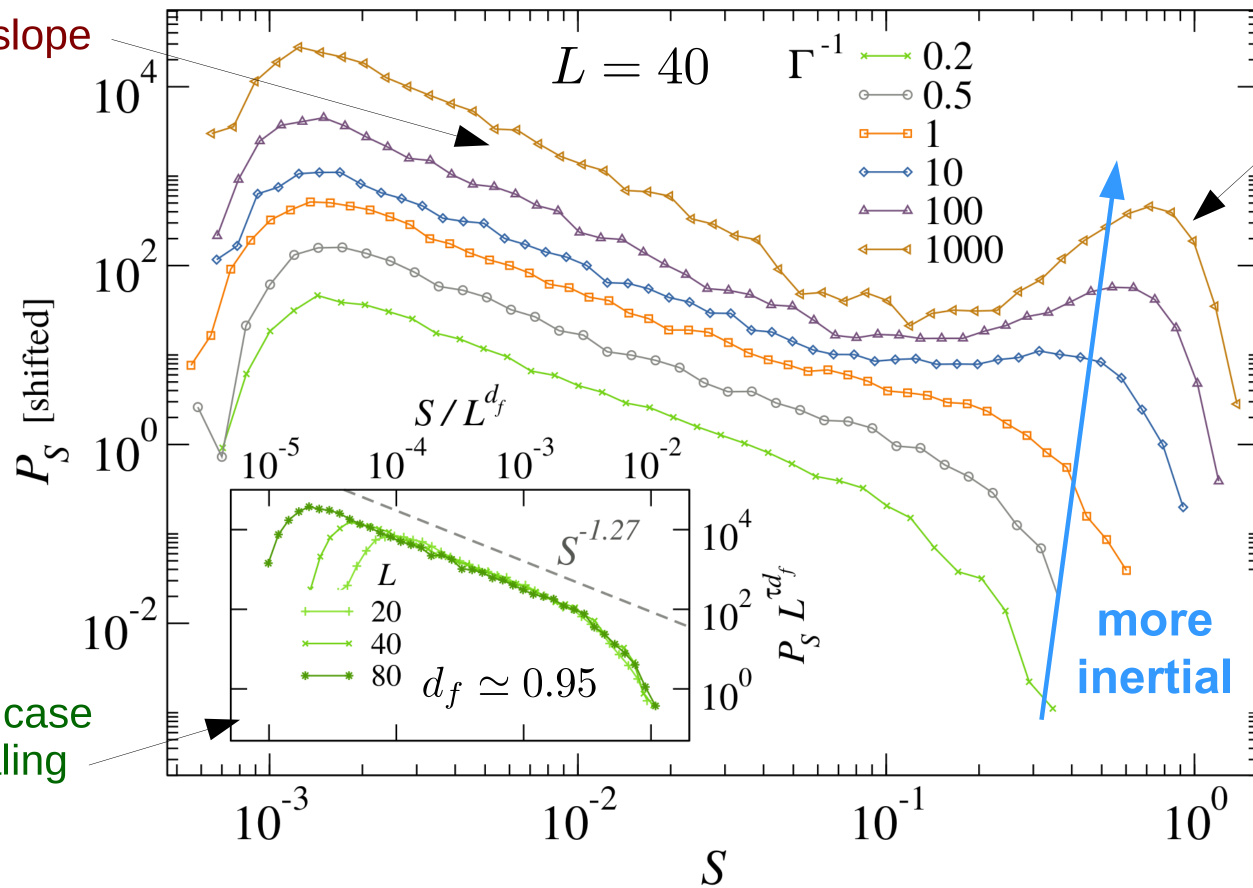
Inertial avalanche size distributions

$$S \equiv \langle \sigma \rangle \Delta\sigma L^d$$

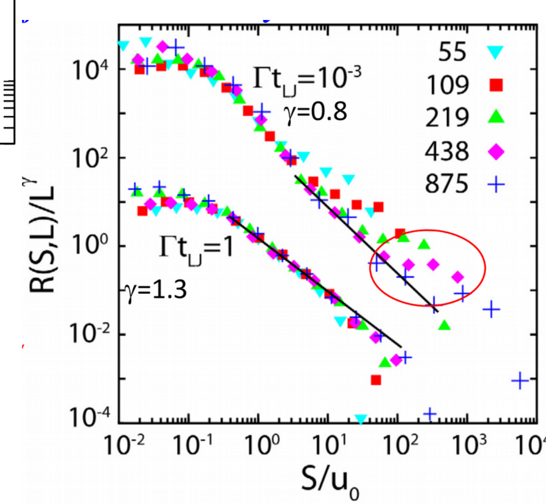
Varying damping

steeper slope

"new" characteristic size



Overdamped case finite size scaling (idem EPM)



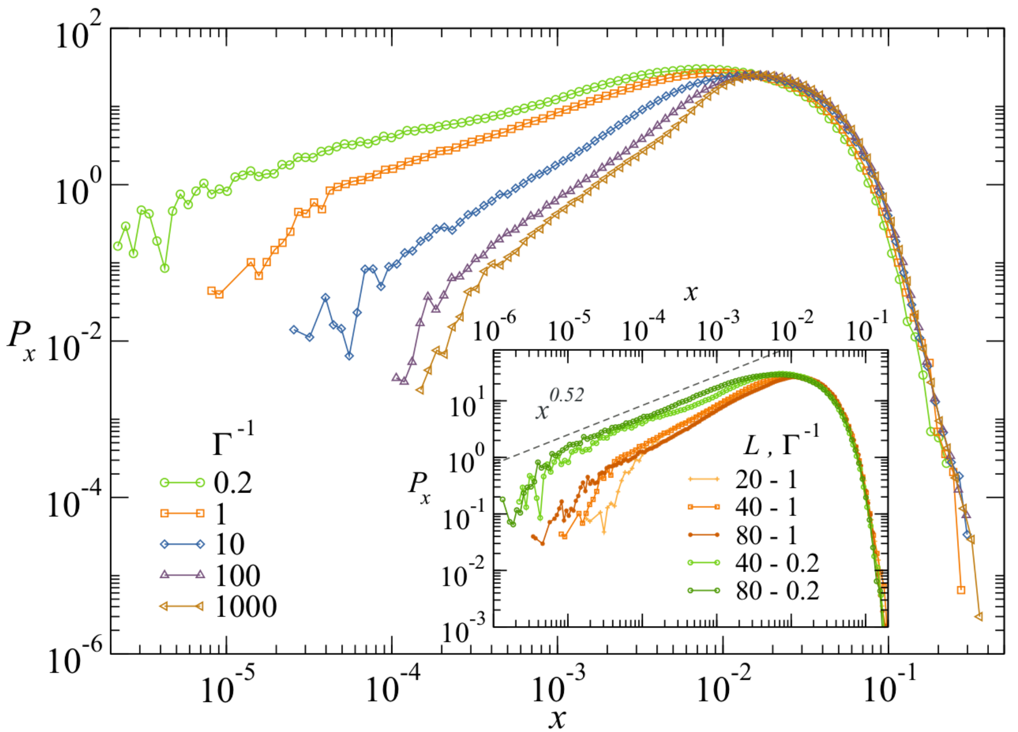
Rather than supporting a new universality class [1], inertia breaks-down the scaling behavior*

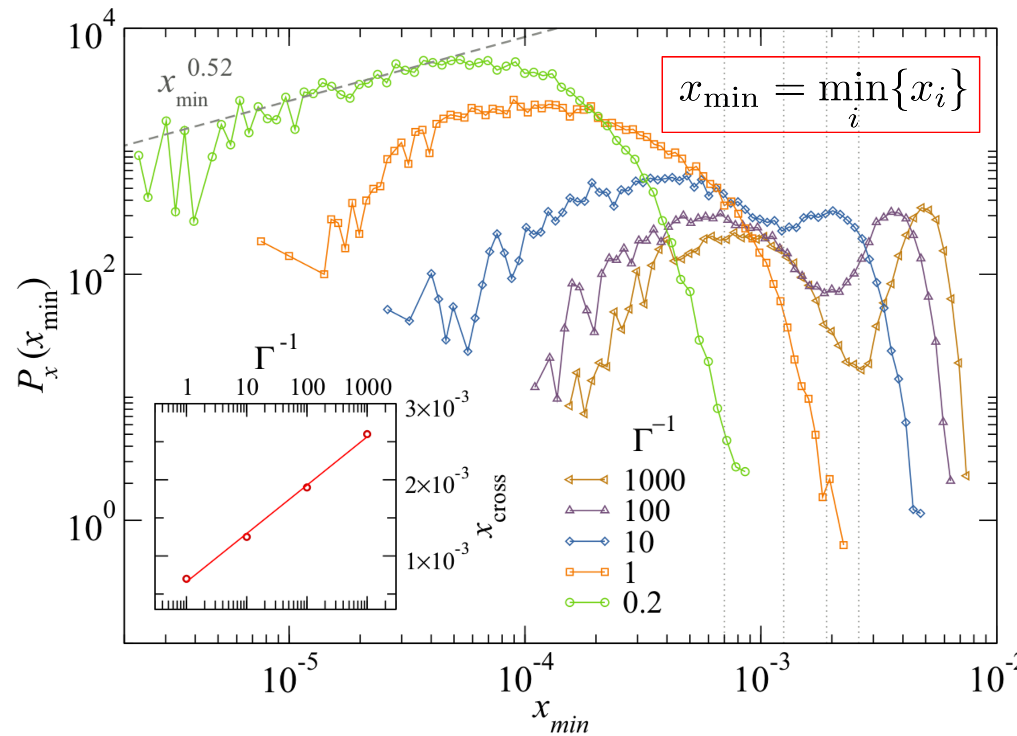
*various examples in SOC, also Burridge-Knopoff model with weakening friction law (Carlson, Langer et al.)

[1] K.M. Salerno & M. Robbins PRE 88, 062206 (2013)

Results

Distances to yielding and minimal distances to yielding distributions





Good **agreement** with EP models overdamped limit. $\theta_{2d} \simeq 0.52$

Increasing inertia we observe a **steeper gap**

The apparent bigger θ as Γ^{-1} increases is a result of the presence of **two kind of events**

$$\tau = 2 - \frac{\theta}{\theta+1}\frac{d}{d_f}$$ does **not hold** anymore

$P(x_{\min})$ displays a **bimodal distribution** for underdamped systems $x_{\min} \lessgtr x_{\min}^{\text{cross}}$

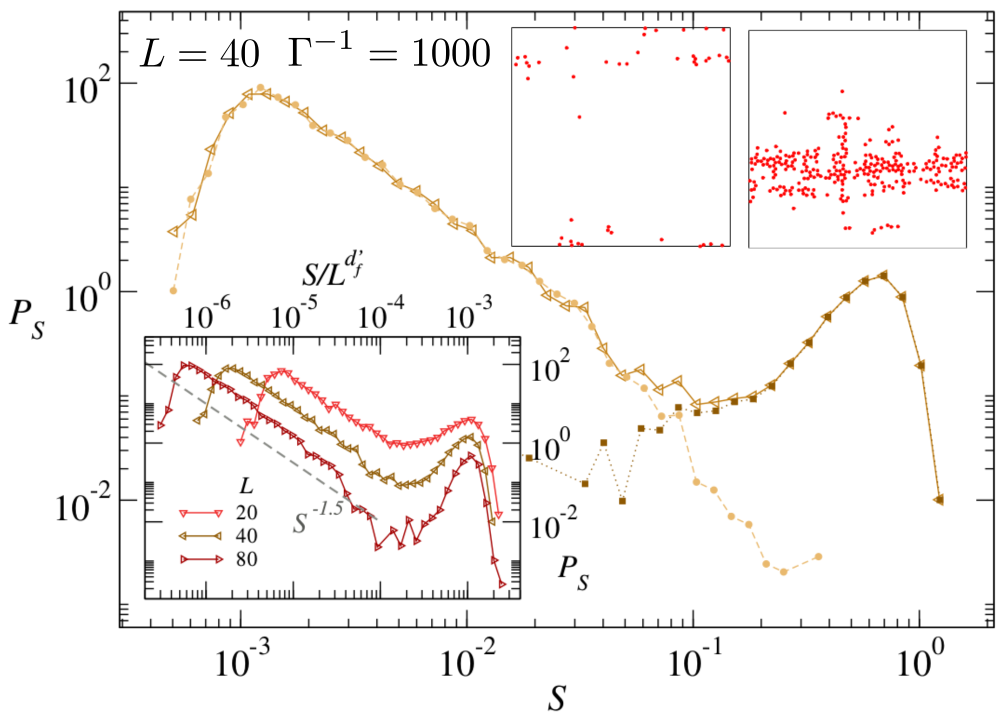
Ansatz:

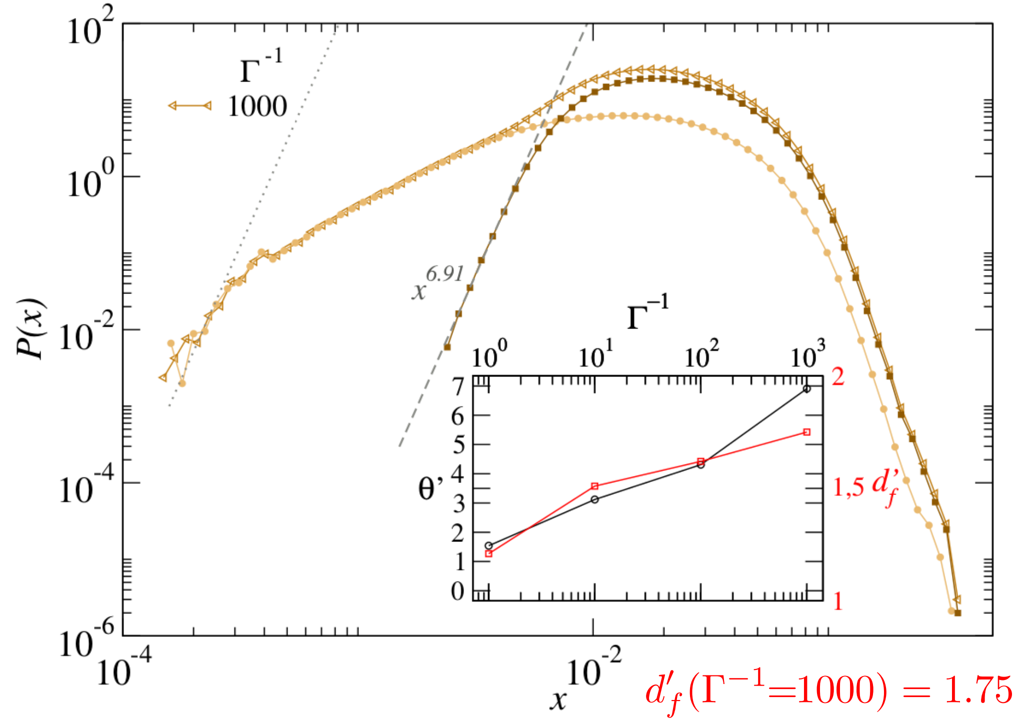
$x_{\min}^{\text{cross}}$ separates two kind of avalanches:

- *massive and **inertial*** (“large” $x_{\min}$)
- *localized and “**overdamped-like”*** (“normal” $x_{\min}$)

Results

Avalanche size and distances to yielding distributions splitting





The splitting **in two contributions** is clear

The **inertial peak** scales with $L^{d_f'}$ $d_f' > d_f$

(consistent with MD**)

"Incipient" shear bands? This is **quasistatic**

New relation holds for the **exponents** related with the **inertial subset**

$$d_f' = d\left(1 - \frac{1}{1+\theta'}\right)$$

Inertia associated with non-monotonicity in the flowcurve*. Same mechanism present here.

*A. Nicolas et al. PRL 116 058303 (2016)
K. Karimi and J.-L. Barrat PRE 93 022904 (2016)

**M. Robbins at KITP Avalanches 2014

Summary 2/2

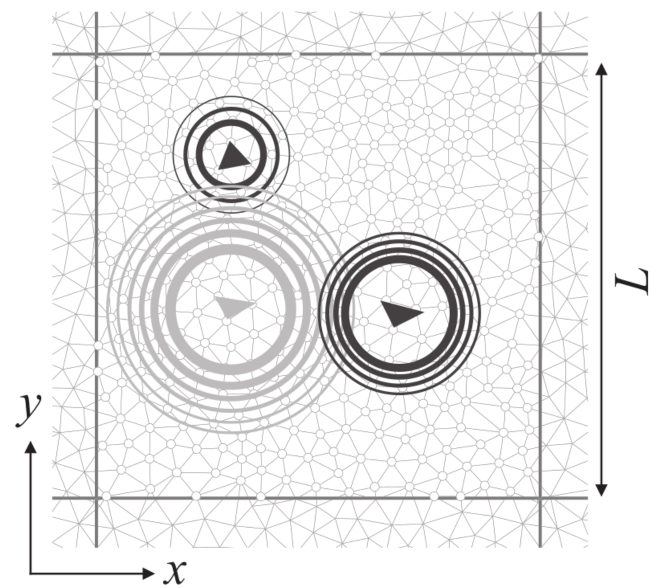
K. Karimi, EEF, J-L Barrat
Phys. Rev. E **95**, 013003 (2017)

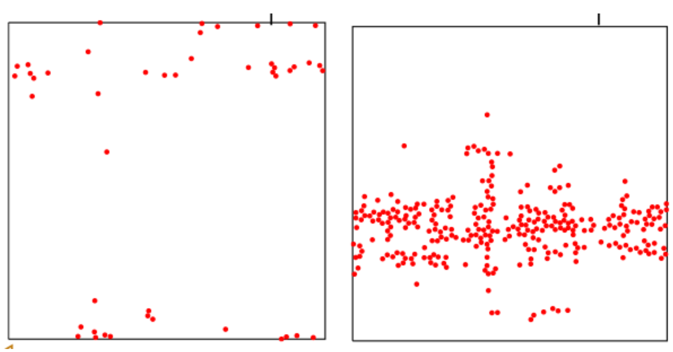
- **Inertia breaks down the scale-free** avalanche statistics and dominates the scaling of **large** avalanches, that show a **larger fractal dimensions** and reminiscence of **shear bands**

- A **power-law distribution** with **damping dependent exponent** is seen for **smaller** avalanches.

- We are able to **discriminate** “inertial” form “overdamped-like” avalanches based on the value of the minimum **distance to threshold** after them.

- In contrast to SOC-depinning models, **d_f being smaller than d** in the overdamped limit of amorphous solids **leaves** a lot of **“room”** for the deployment of inertial avalanches when damping is decreased (the bump both **grows <u>and</u> moves** to the right).





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European Research Council

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Stefano Zapperi

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CIMENT - UGA

Thanks !

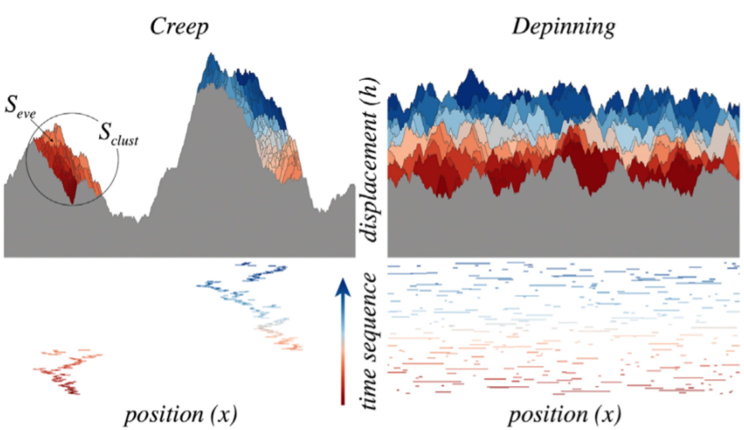
www.ezequielferrero.com

References:

Phys. Rev. Lett. **116** 065501 (2016)
Phys. Rev. E **95**, 013003 (2017)

“Spatiotemporal Patterns in Ultraslow Domain Wall Creep Dynamics”

Ezequiel E. Ferrero,
Laura Foini,
Thierry Giamarchi,
Alejandro B. Kolton,
Alberto Rosso
Phys. Rev. Lett.
118, 147208 (2017)



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