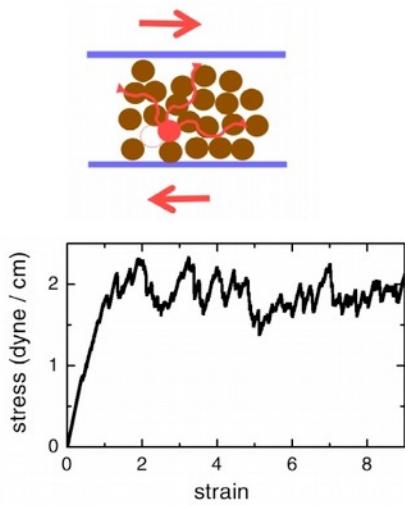


# Avalanches in amorphous solids when approaching (or leaving) the yielding transition

Ezequiel Ferrero

Université Grenoble Alpes



MMM Dijon, 11/10/2016

*Driving Rate Dependence of Avalanche Statistics and Shapes at the Yielding Transition*  
Chen Liu, EEF, Francesco Puosi, Jean-Louis Barrat, Kirsten Martens  
*Phys. Rev. Lett.* **116** 065501 (2016)

# Avalanches: mean-field approaches

**Fully-connected** network of  $N$  yield stress **blocks**

1) We (somehow) **push** blocks towards instability

2) Block  $m$  reaches the threshold ( $\sigma_c = 1 \quad \forall i$ )

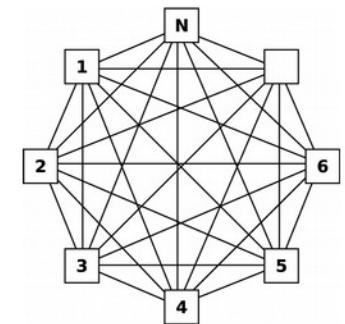
- the stress in  $m$  drops by a (random) amount
- all other blocks receive stress “kicks”

3) We repeat (2) while blocks yield, “avalanche size” is

4) We resume from (1)

$$\sigma_i \quad i = 1, \dots, N$$

$$\sigma_i \rightarrow \sigma_i + \delta\sigma$$



$$\sigma_m \rightarrow \sigma_m - u(1 + k) \quad \frac{u}{k} \sim \frac{1}{N}$$

$$\sigma_i \rightarrow \sigma_i + \frac{u}{N} + \frac{\eta}{\sqrt{N}}$$

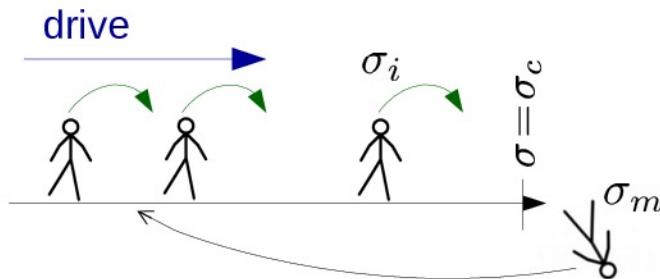
$\eta$  Gaussian rn,  $\langle \eta \rangle = 0$ , variance  $\omega$

$$S = \sum_m u_m$$

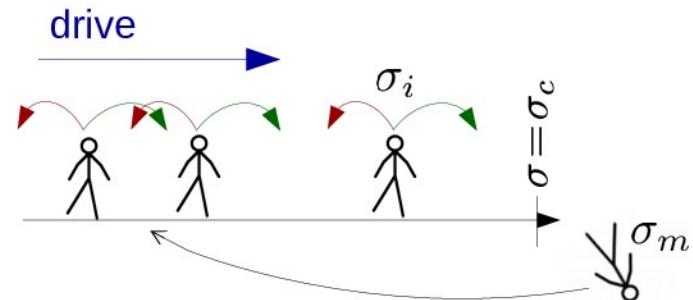
$$\omega = 0 \quad P(\eta) = \delta(\eta)$$

$$\omega > 0 \quad P(\eta) \sim e^{-\eta^2/2\omega}$$

All “kicks” are positive (depinning case)



“kicks” are positive and negative (yielding case)



# Avalanches: mean-field approaches (simulations)

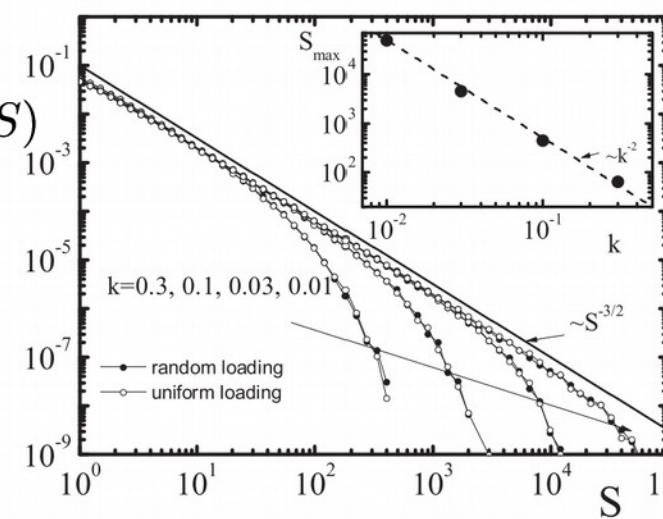
E. Jagla PRE **92** 042135 (2015)

$$\sigma_m \rightarrow \sigma_m - u(1+k)$$

$$P(S) \sim S^{-\tau} f(S/S_{\max}(k))$$

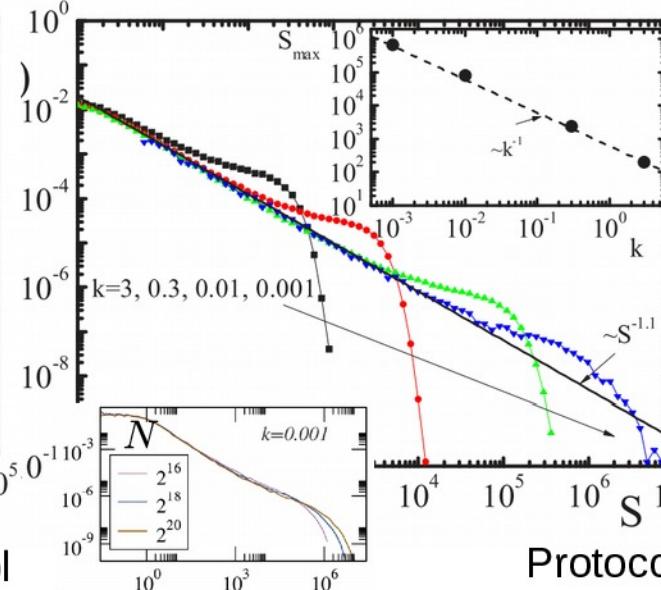
critical point:  $k \rightarrow 0$

Depinning case  $\omega = 0$



independent on loading protocol

Yielding case  $\omega > 0$



Protocol dependent!!

$$\tau = 1.5$$

$$S_{\max} \sim k^{-1/(2-\tau)}$$

Uniform loading

$$\tau \simeq 1.1$$

$$S_{\max} \sim (\sqrt{N}/k)^{1/(2-\tau)}$$

Random loading

$$\tau = 1.5$$

$$S_{\max} \sim k^{-1/(2-\tau)}$$

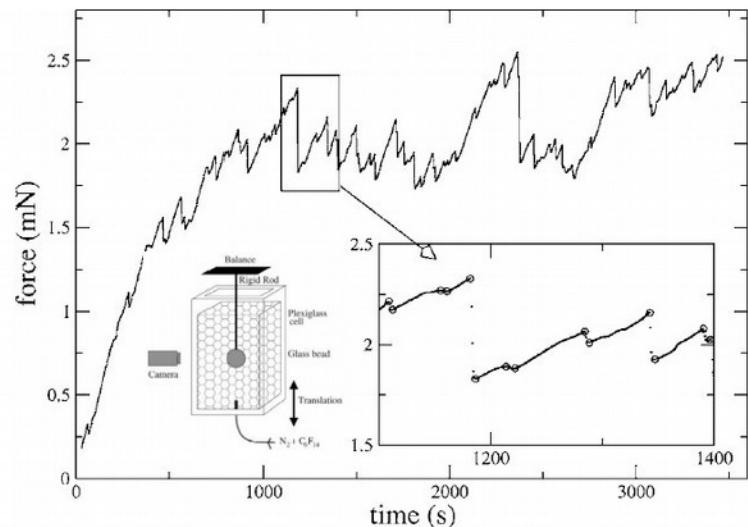
HL-like model (which catches the “non-positive” nature of the **Eshelby propagator**) yields an exponent **different from depinning**. Yet, random triggering restore a constant rate stochastic process for instability and  $\tau=3/2$ .

# Basic Phenomenology

## Local rearrangements

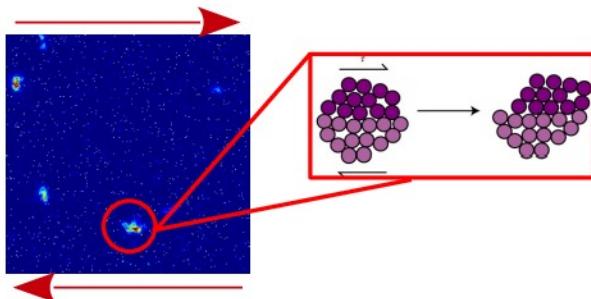


“jerky” aspect of the stress response



I. Cantat and O. Pitois *Phys. Fluids* **18** 083302 (2006)

well identified, localized “**plastic events**”

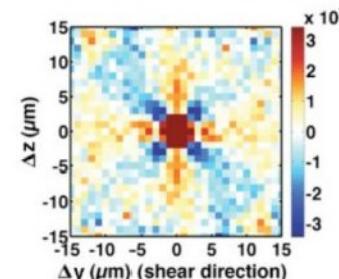


“*plastic event*” = “*rearrangement*” = “*shear transformation*”

A. Nicolas et. al EPJE 37 50 (2014), Argon and Kuo Mat. Sci. Eng. **39** 101 (1979)

## Medium elastic response

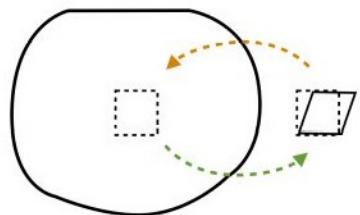
correlations of local strain  
(sheared colloidal glass)



Jensen et al, *PRE* **90**, 042305 (2014) Desmond and Weeks, *PRL* **115**, 098302 (2015)

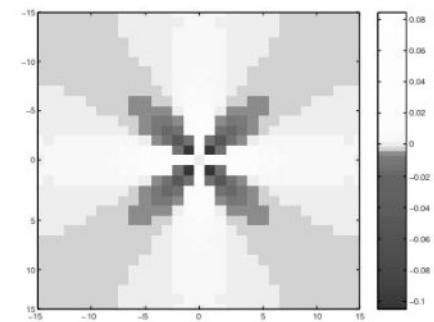
*Continuum mechanics:*

elastic response to a deformed inclusion



“Eshelby” propagator for the strain (stress) redistribution

$$G^{2D}(r, \theta) = \frac{1}{\pi r^2} \cos(4\theta)$$

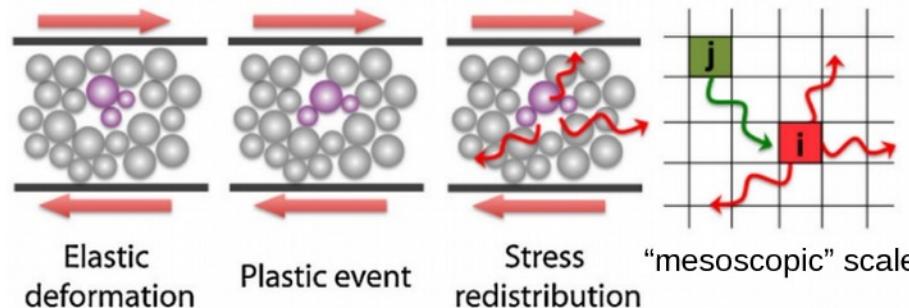


J.D. Eshelby Proc.Roy.Soc. A **241** 376 (1957)

Picard et al. EPJE **15** 371 (2004)

# Coarse-grained elasto-plastic model

Fig. credit:  
Bocquet *et al.* PRL 103, 036001 (2009)



## Simplifications:

- Scalar
- Athermal
- Overdamped
- p.b.c.

**Scalar** stress field ( $\sigma \equiv \sigma_{xy}$ ) in a grid, representing the stress in **each block**

$$\partial_t \sigma(i, t) = \underbrace{\mu \dot{\gamma}^{\text{ext}}}_{\text{external strain-rate}} - \underbrace{g_0 n(i, t) \frac{\sigma(i, t)}{\tau}}_{\text{local plastic yield}} + \underbrace{\sum_{j \neq i} G(i, j) n(j, t) \frac{\sigma(j, t)}{\tau}}_{\text{"mechanical noise" due to plastic activity}}$$

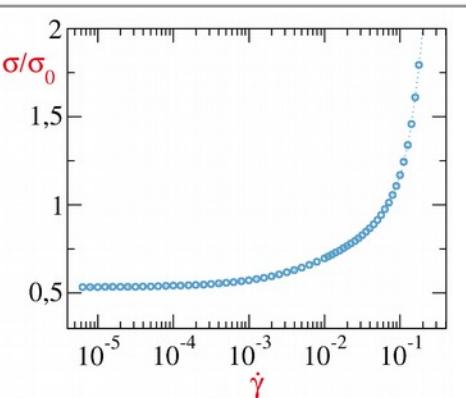
## Eshelby propagator

$$G_{2d}(i, j) = \cos(4\theta_{ij}) / \pi r^2 \quad r = |\mathbf{r}_i - \mathbf{r}_j|$$

Describes a **yield-stress material**

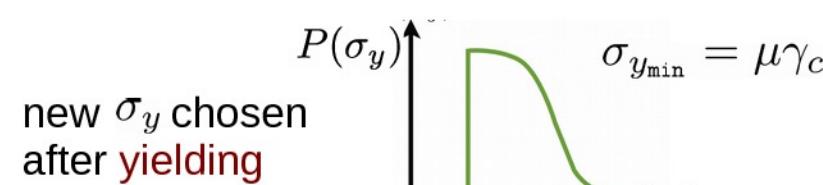
$$\langle \sigma \rangle = \sigma_y + A \dot{\gamma}^n$$

Herschel–Bulkley curve



+ dynamical **rules** for a local “**state variable**”

$$n_i : \begin{cases} 0 \rightarrow 1 \text{ when } \sigma_i > \sigma_{y_i} \\ 1 \rightarrow 0 \text{ when } \int dt' |\dot{\gamma}_i^{\text{tot}}(t')| \geq \gamma_c \end{cases}$$

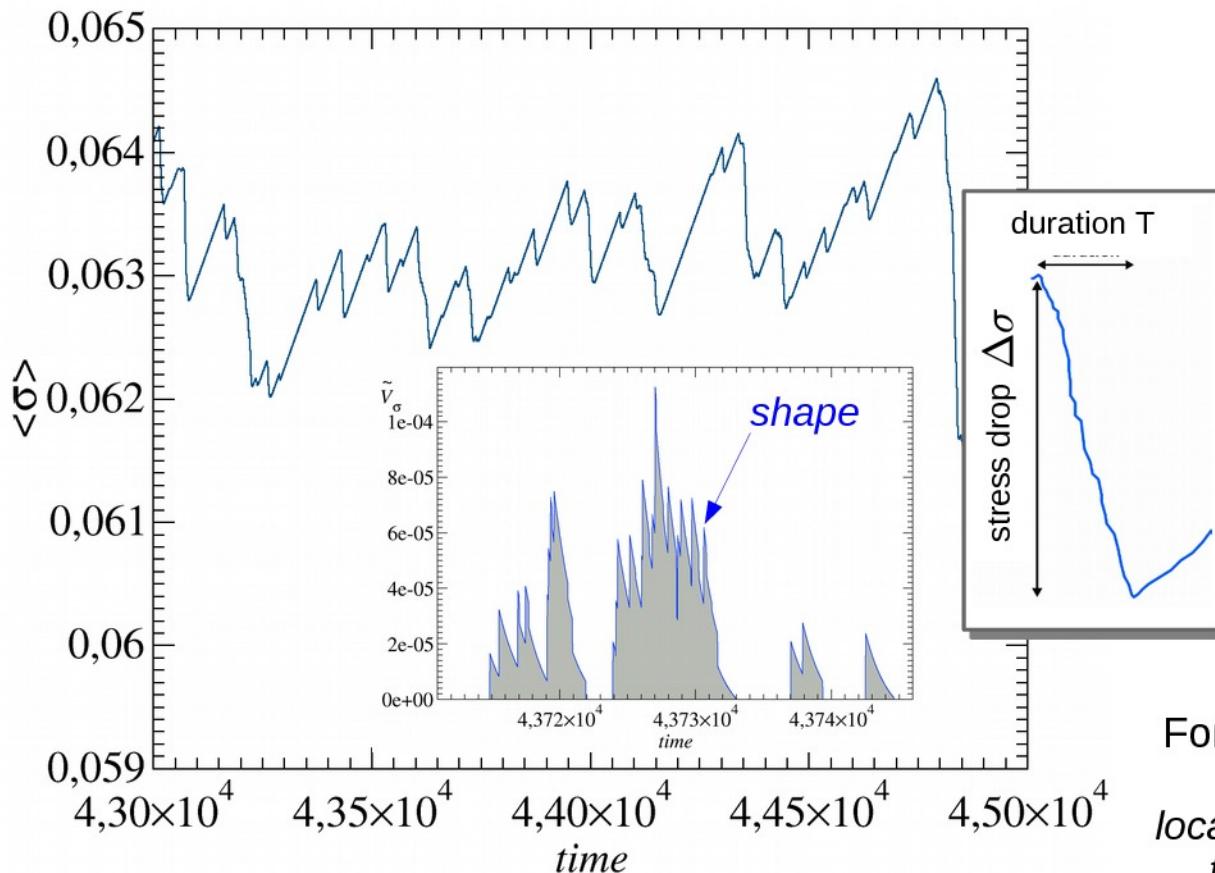


# Avalanches: Methods

For different imposed strain rates...

## Observables:

For each event (stress-drop  $\Delta\sigma$ ) we compute:



$$duration \quad T = t_{\text{end}} - t_{\text{start}}$$

$$size \quad S \equiv L^d \Delta\sigma$$

$$\tilde{V}_\sigma(t) = \left[ -\frac{d\langle\sigma\rangle(t)}{dt} \right]_{t_{\text{end}}}^{t_{\text{start}}}$$

For several configurations in time

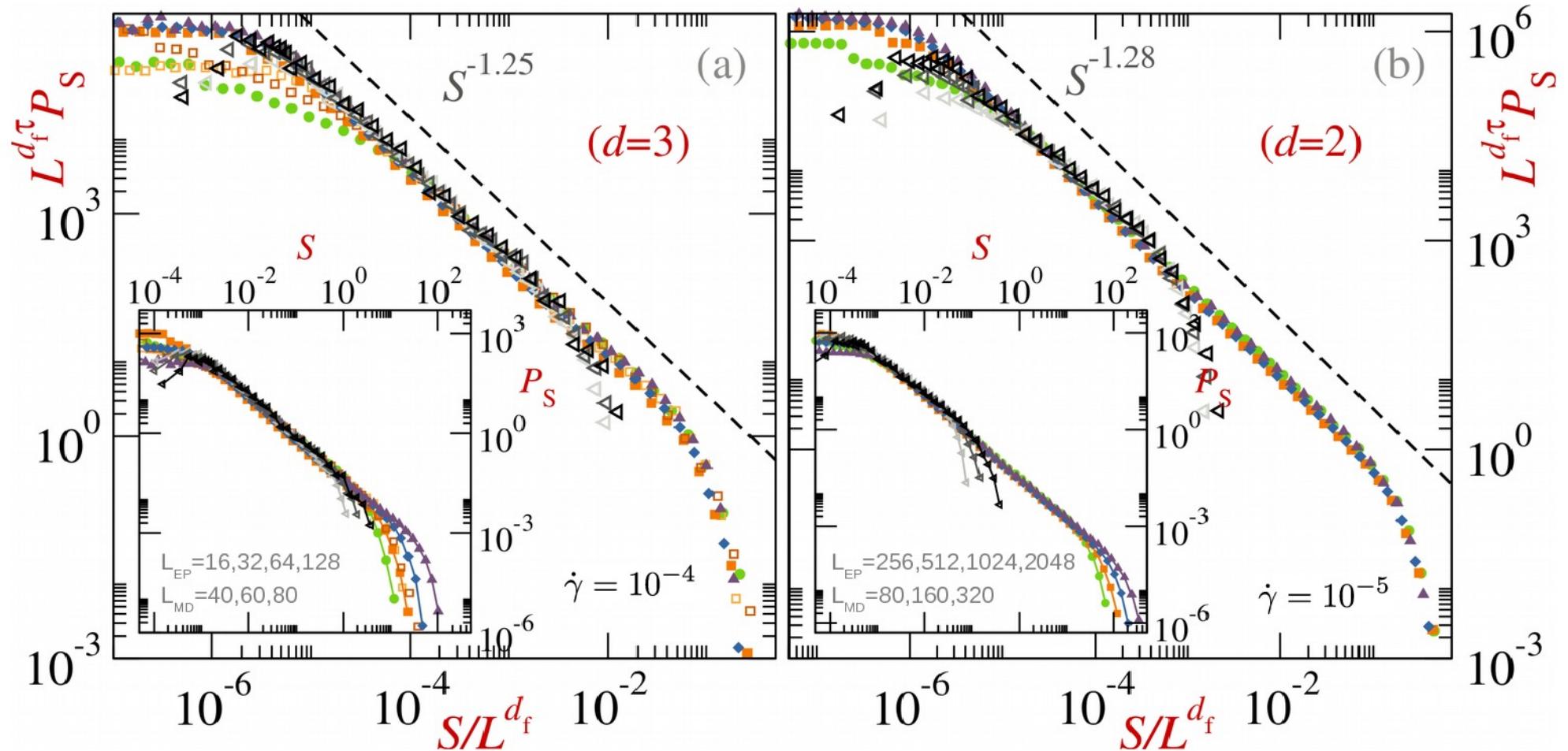
local distances  
to threshold

$$\{x\}, \quad x_i \equiv \sigma_i^y - \sigma_i$$

# Results

## Stress drop size distribution at very low shear rates

...for different system sizes, comparing with quasistatic MD simulations (grayscale triangles)



$$\tau_{3D} \simeq 1.25$$

$$d_f^{3D} \simeq 1.3$$

$$P_S \propto S^{-\tau} f(S/S_c), \quad S_c \simeq L^{d_f}$$

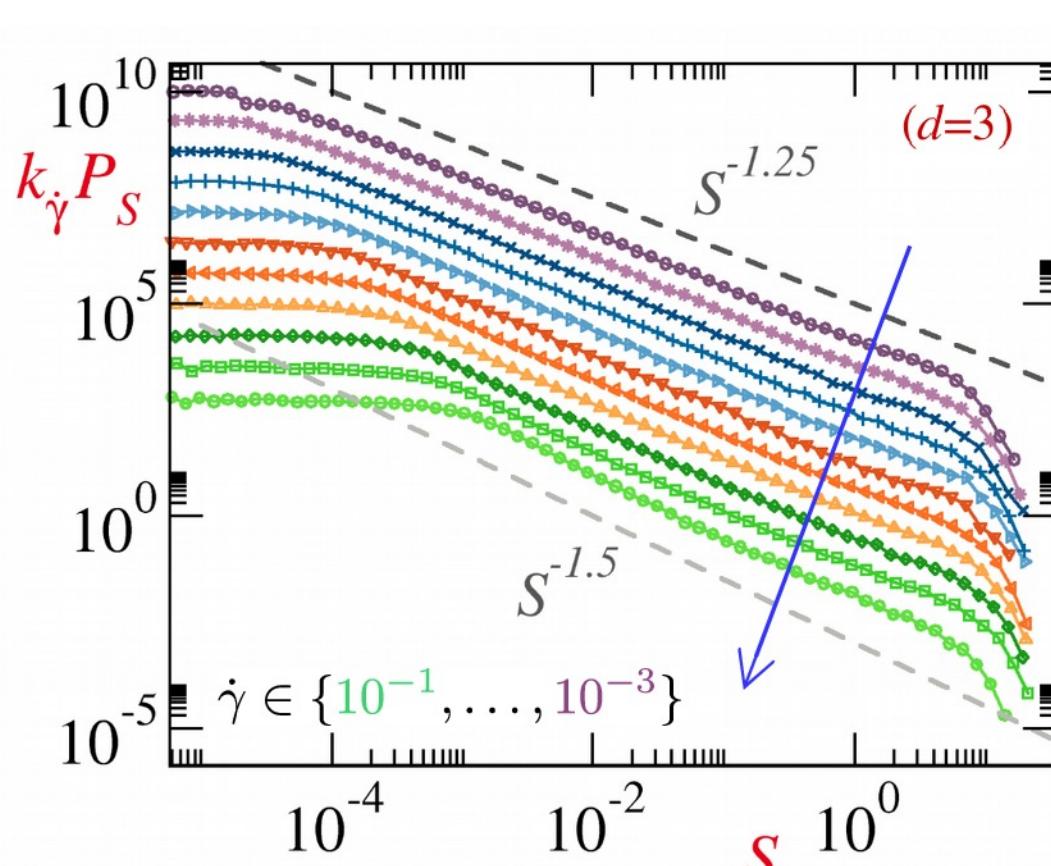
$$\tau_{2D} \simeq 1.28$$

$$d_f^{2D} \simeq 0.9$$

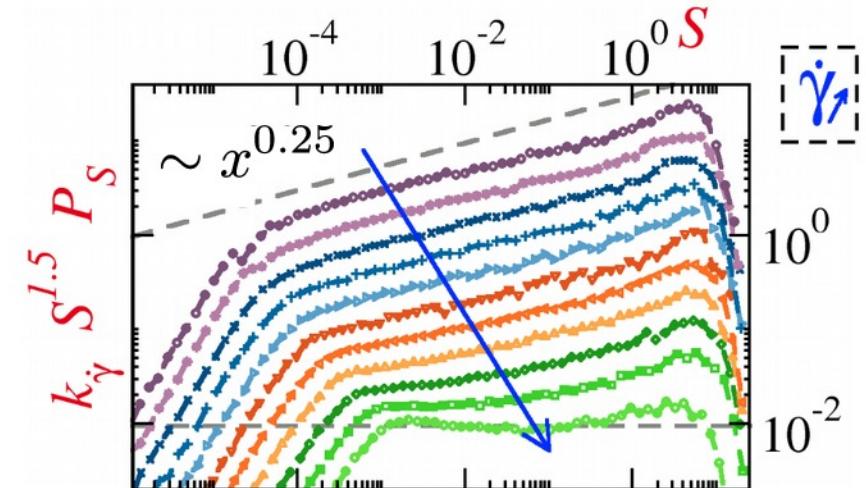
scaling factor  $d_f$ : "fractal dimension". *Slip-line* avalanche geometry

# Results

## Size distributions and crossover to mean-field behavior



(curves arbitrarily shifted by  $k_\gamma$ )



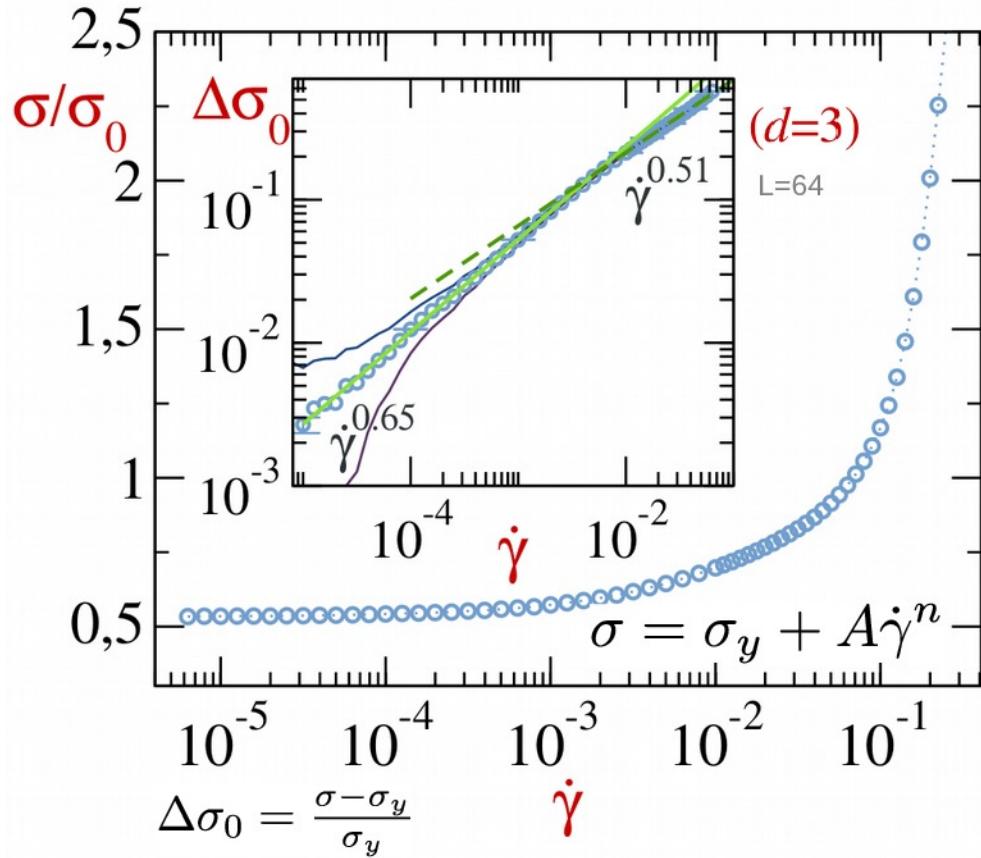
- Large strain-rates “**randomizes**” the stress signal, by overlapping **uncorrelated** plastic activity.
- Crossover to “**random triggering**” (or depinning) mean-field exponent when we **go away** from the yielding point

$$\tau : 1.25 \rightarrow 1.5$$

Be  $\xi^d$  the size of a “correlated event”, with  $\xi \sim |\langle \sigma \rangle - \sigma|^{-\nu} \sim \dot{\gamma}^{-\nu/\beta}$ ,  $\beta = 1/n$   
 In this regime, many events may “fit” in  $L^d$ .  $\Delta\sigma$  results from this superposition.  
 $S \equiv \Delta\sigma L^d$  cutoff is controlled by  $L$

# Results

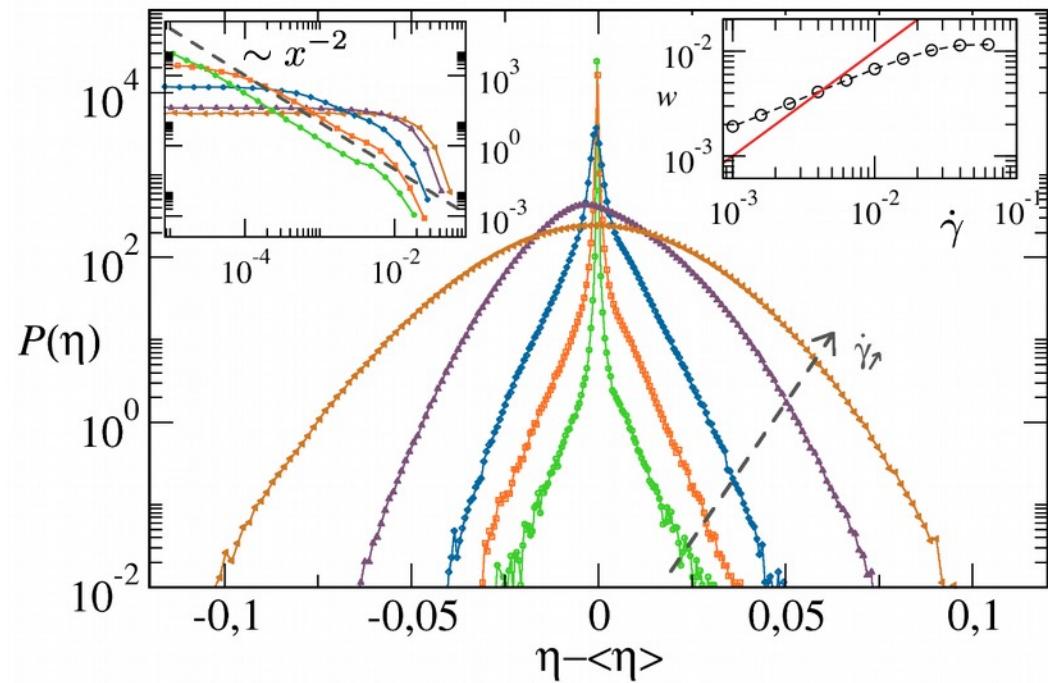
Flow-curve and crossover to mean-field “randomized” behavior



The “yielding transition”     $\dot{\gamma} \sim (\sigma - \sigma_y)^\beta$   
 $\beta = 1/n > 1$

- $\beta$  crosses over toward the Hébraud-Lequeux mean-field prediction when  $\dot{\gamma}$  increases.

$$\beta \simeq 1.54 \rightarrow 2$$



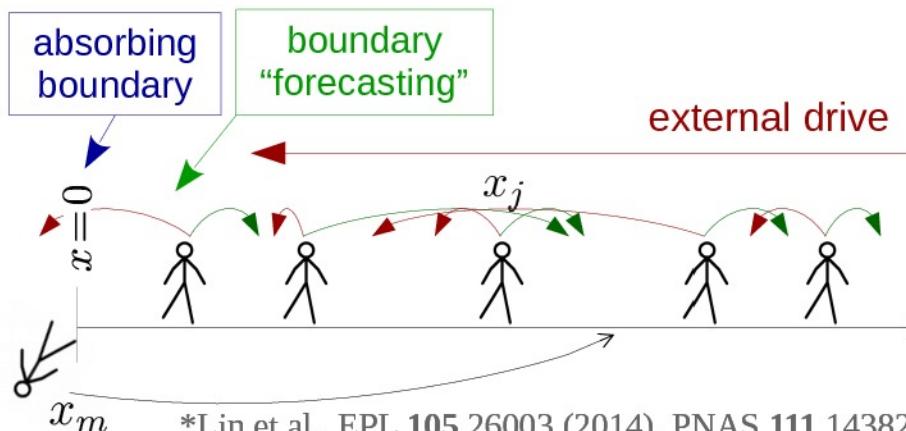
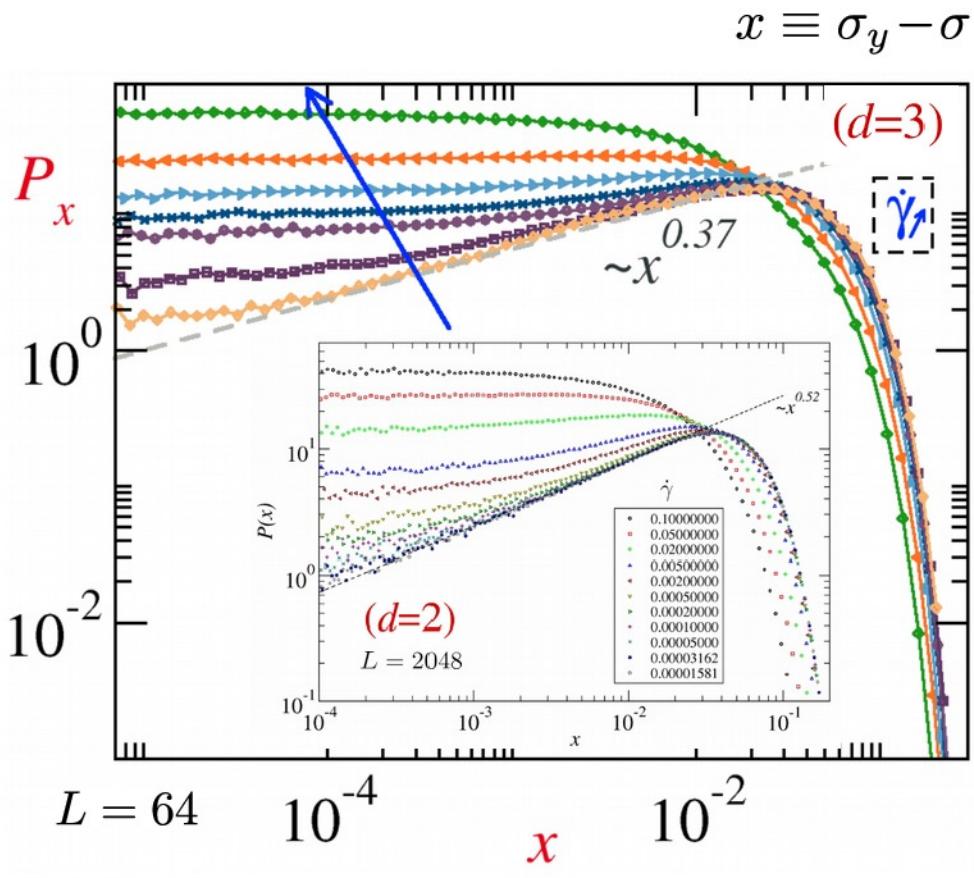
“Mechanical noise”     $\eta_i = \sum_{j \neq i} G_{ij} n_j(t) \frac{\sigma_j(t)}{\tau}$

- At large rates, noise distribution turns Gaussian  
 $\rightarrow$  **loss of non-trivial correlations**
- Variance grow slower than linear with  $\dot{\gamma}$   
 $\rightarrow$  **drift dominates when  $\dot{\gamma} \gg 1$**

$$\partial_t \sigma_i = \mu \dot{\gamma} - g_0 n_i \frac{\sigma_i}{\tau} + \sum_{j \neq i} G_{ij} n_j \frac{\sigma_j}{\tau}$$

# Results

Distribution of local distances to threshold (or “density of shear transformations”)



\*Lin et al., EPL 105 26003 (2014), PNAS 111 14382 (2014)

We expect: “marginal stability” pseudo-gap  
(M. Wyart and co.)

$$P_x \sim x^\theta$$

$$\theta > 0$$

$$\begin{aligned} \theta_{2D}^{qs} &\simeq 0.57 \\ \theta_{3D}^{qs} &\simeq 0.35 \end{aligned}$$

We observe:

$$\text{At } \dot{\gamma} \rightarrow 0 \quad \theta_{2D} \simeq 0.52 \quad \theta_{3D} \simeq 0.37$$

$$\text{When } \dot{\gamma} \gg 0 \quad \theta \rightarrow \theta^{\text{dep}} = 0$$

depinning yielding

$$\beta \leq 1$$

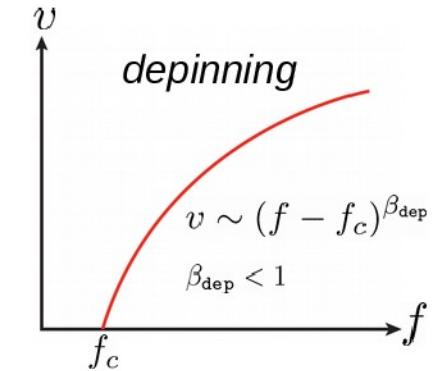
$$d_f \geq d$$

$$\theta = 0$$

$$\beta > 1$$

$$d_f < d$$

$$\theta > 0$$

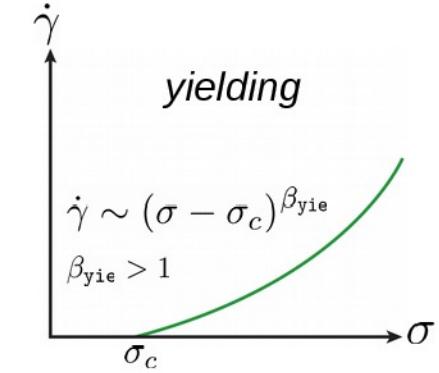


Scaling relations:

$$\beta = \nu(d - d_f + z)$$

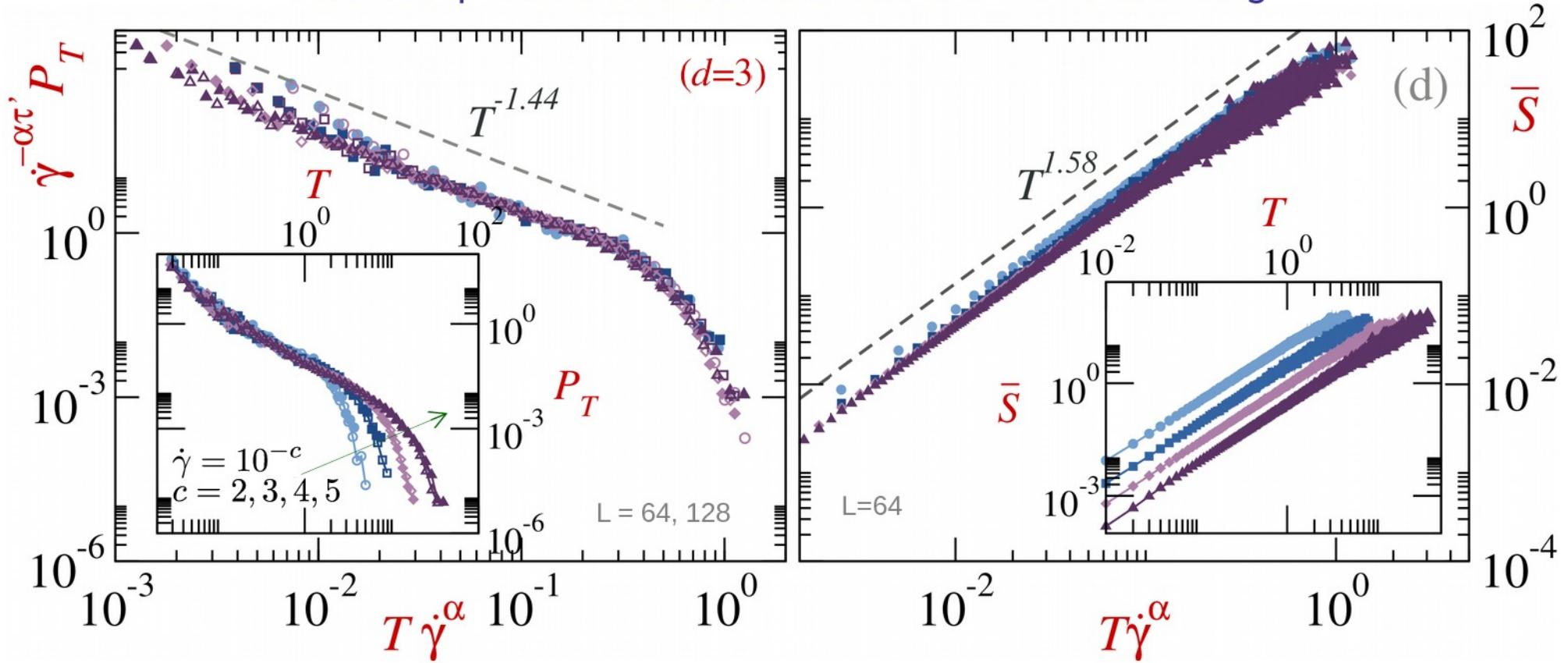
$$\nu = 1/(d - d_f)$$

$$\tau = 2 - \frac{\theta}{\theta + 1} \frac{d}{d_f}$$



# Results

## Stress drop duration distribution and size-duration scaling



$$P_T \propto S^{-\tau'} g(T/T_c), \quad T_c \simeq \dot{\gamma}^{-\alpha}$$

$\tau'_{3D} \simeq 1.44$	$\alpha_{3D} \simeq 0.30$
$\tau'_{2D} \simeq 1.41$	$\alpha_{2D} \simeq 0.38$

Who is  $\alpha$ ?

$$T \sim \xi^z \sim \dot{\gamma}^{-\nu z / \beta} \Rightarrow \alpha \equiv \nu z / \beta = 1 - 1/\beta$$

$$\xi \sim |\langle \sigma \rangle - \sigma_c|^{-\nu} \quad T_c \sim \dot{\gamma}^{n-1}$$

We expect:  $S \sim T^\delta \quad \delta = d_f/z$

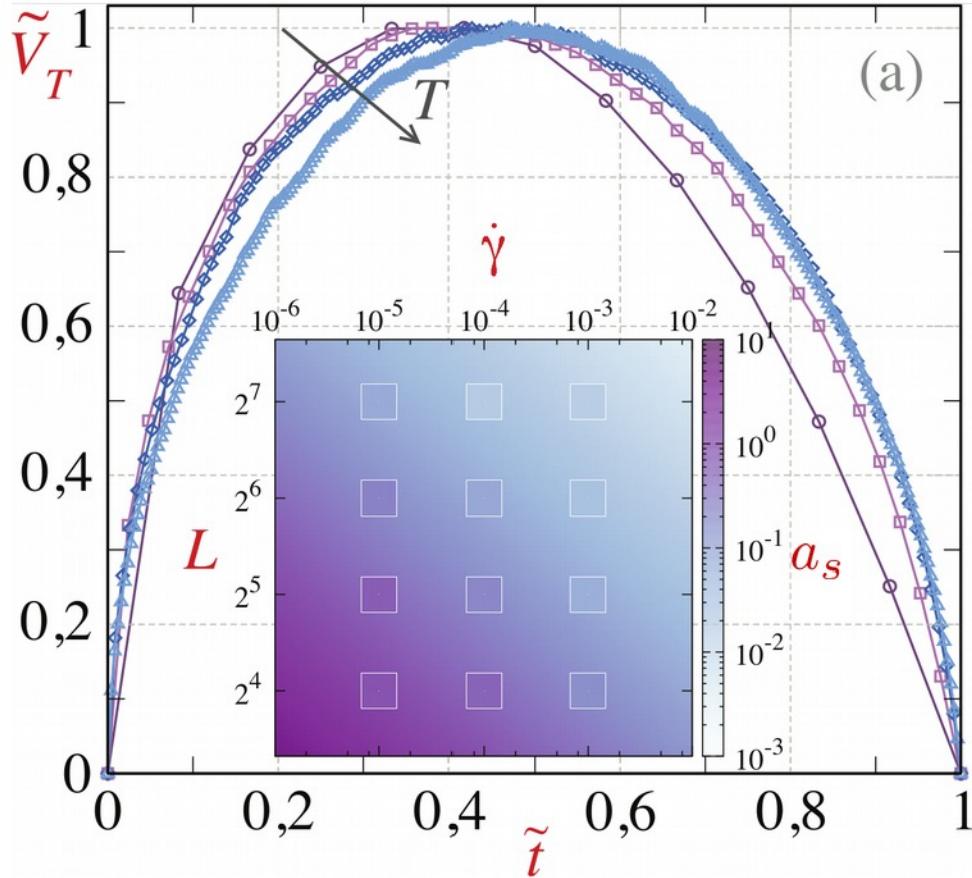
We observe:  $S \simeq C(L, \dot{\gamma}) T^\delta$

$$\frac{S}{L^{d_f}} \sim \left( \frac{T}{\dot{\gamma}^{-\alpha}} \right)^\delta \quad \delta \simeq 1.58$$

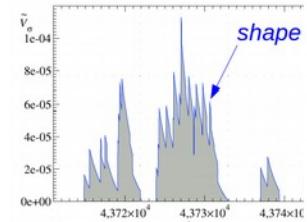
Again, exponents differ from MF depinning

# Results

## Stress drop shapes



**Recall:**  $\tilde{V}_\sigma(t) = \left[ -\frac{d\langle\sigma\rangle(t)}{dt} \right]_{t_{\text{end}}}^{t_{\text{start}}}$



**Normalized shape** for a drop of duration  $T$ :

$$\tilde{V}_T(\tilde{t}) = V_T(t)/ \max_t(V_T(t)) \quad \tilde{t} = t/T$$

**Fitting function\*:**

$$\tilde{V}_T(\tilde{t}) \propto B(\tilde{t}(1-\tilde{t}))^c(1 - a_s(\tilde{t} - 0.5))$$

$$B \sim T^c \quad c = \delta - 1 \quad \text{holds} \quad B \sim T^{0.6}$$

*Inset: “asymmetry” parameter*

- Drops of **short durations** show a clearly **asymmetric** shape
- For **large  $T$**  stress drops shapes become more **symmetric**.
- **Superposition** of “individual” avalanches due to **finite strain-rate**.

# Summary

C. Liu, E.E.F., F. Puosi, J.-L. Barrat, K. Martens  
*Phys. Rev. Lett.* **116** 065501 (2016)

- We studied the **avalanche statistics** close and departing from the **yielding transition**.
- Our results reinforce the idea of a **non-mean field universality class** for  $d=2,3$  or, at least, a universality class clearly **different from** mean-field **depinning**.
- **Departing from** the **yielding** point, at relatively large **strain rates**, the rise of many independent regions with yielding activity draw **exponents** and **avalanche shapes towards mean-field expectations**.
- A **finite strain rate** also enters in the **scaling description** of avalanches duration, the **flow-curve exponent** controls the duration cutoff and the size-duration scaling.

## Financial support:

"GLASSDEF"  
No. ADG20110209  
Jean-Louis Barrat



European Research Council

## Computing resources:

Froggy hybrid cluster  
CIMENT - UGA



# Thanks !

[www.ezequielferrero.com](http://www.ezequielferrero.com)

